



MEERUT INSTITUTE OF ENGINEERING AND TECHNOLOGY
NH-58, Delhi-Roorkee Highway, Baghpat Road, Meerut - 250 005 (U.P.)

Roll No.

Pre-University Test (PUT) : ODD Semester 2022 - 23

Course/Branches: B Tech (CSE/IT/CS-IT/DS/IOT/AI/AIML)

Semester: III

Subject Name: Discrete Structures & Theory of Logic

Max. Marks: 100

Subject Code: KCS - 303

Time Allowed: 180 Minutes

- CO - 1: apply concepts related to sets, functions, relations, and also some methods of proof, in the study of fundamental discrete structures;
CO - 2: apply concepts related to groups, rings, and fields, in the study of some topics concerning computer science;
CO - 3: apply concepts related to posets, lattices, and Boolean algebras, in the study of some topics concerning computer science;
CO - 4: apply concepts related to propositional calculus and first order logic in the study of some topics concerning computer science;
CO - 5: apply basic graph theory, trees, and some simple combinatorial techniques, in the study of some topics concerning computer science.

Section - A # 20 Marks

Attempt ALL the questions. Each Question is of 2 marks ($10 \times 2 = 20$ marks)

Q 1 Attempt ALL the questions.

| Part | CO | Question Description | BT |
|------|-----|---|-------|
| A | CO1 | Find the power set of the set $\{a, \{a\}\}$. | K_3 |
| B | CO1 | Let A be a set with n elements. How many reflexive and symmetric relations are there on the set A ? | K_3 |
| C | CO2 | Show that every cyclic group is Abelian. | K_3 |
| D | CO2 | Define ring and field. | K_1 |
| E | CO3 | Differentiate between distributive and complemented lattices. | K_2 |
| F | CO3 | Explain why a lattice with 5 elements cannot be a Boolean algebra? | K_2 |
| G | CO4 | Write the inverse and contrapositive of the statement: "If today is Monday or sunny day then I will go to college". | K_3 |
| H | CO4 | Write the statement "All flowers are beautiful" in symbolic form. | K_3 |
| I | CO5 | Explain why there is no graph with degree sequence $(4, 3, 3, 2, 1)$? | K_3 |
| J | CO5 | State generalised pigeonhole principle. | K_1 |

Section - B # 30 Marks

Attempt ALL the questions. Each Question is of 6 marks ($5 \times 6 = 30$ marks)

Q 2 (CO1) Let $A = \{1, 2, 3, \dots, 6, 7\}$. Show that $R = \{(x, y) \in X \times X \mid x - y \text{ is divisible by } 3\}$ is an equivalence relation on A ? Also, draw the digraph of R . (K_3)

OR

State principle of mathematical induction. Prove that, for all $n \geq 2$, (K_3)

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} \geq \sqrt{n}.$$

Q 3 (CO2) Define group. Show that the set $\{1, 2, 3, 4, 5\}$ is not a group with respect to operations $+_6$ (addition modulo 6) and \times_6 (multiplication modulo 6). (K₃)

OR
State and prove Lagrange theorem for finite groups, and hence deduce that every group of prime order is cyclic. (K₃)

Q 4 (CO3) Define a lattice. Let L be a lattice, and $a, b, c, d \in L$, be such that $a \leq b$ and $c \leq d$. Show that $a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$. (K₃)

OR
Find the minimal SOP expressions of the Boolean function $F(a, b, c, d) = \sum (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11)$. Also draw the logic circuits for the simplified expression so obtained. (K₃)

Q 5 (CO4) Show that the premises (i) it is not sunny this afternoon and it is colder than yesterday; (ii) we will go swimming only if it is sunny; (iii) if we don't go swimming, then we will take a canoe trip; and, (iv) if we take a canoe trip, then we will be home by sunset, lead to the conclusion "We will be home by sunset". (K₃)

OR
Translate the following statements in quantified expressions: (i) All students need financial aid; (ii) If a person is female, and is a parent, then this person is someone's mother.; (iii) Not all birds can fly; (iv) Nobody loves everybody; (v) All integers are either even or odd. (K₃)

Q 6 (CO5) Define planar graph. Prove that for any connected planar graph $n - e + r = 2$, where n, e, r are respectively the number of vertices, edges, and regions of the graph. (K₃)

OR
Find number of integers between 1 and 500 not divisible by any of four integers 2, 3, 5 or 7. (K₃)

Section - C # 50 Marks

Attempt ALL the questions. Each Question is of 10 marks ($5 \times 10 = 50$ marks)

Q 7 (CO1) Attempt any TWO questions. Each question is of 5 marks.

(a) Define transitive closure of a relation R given on a set A . Find the transitive closure of the relation $R = \{(1, 2), (2, 3), (3, 1)\}$ on the set $A = \{1, 2, 3\}$, by using Warshall's algorithm. (K₃)

(b) Consider the functions $f: \mathbb{R} \rightarrow \mathbb{R}$, $g: \mathbb{R} \rightarrow \mathbb{R}$, and $h: \mathbb{R} \rightarrow \mathbb{R}$ respectively defined by

$$f(x) = x^3 - 4x, \quad g(x) = \frac{1}{x^2 + 1}, \quad h(x) = x^4.$$

Find the compositions (i) $f \circ g \circ h$ and (ii) $h \circ g \circ f$.

OR Prove that, for all $n \geq 1$, $11^{n+1} + 12^{2n-1}$ is divisible by 133. (K₃)

Q 8 (CO2) Attempt any TWO questions. Each question is of 5 marks. (K₃)

(a) Let G be a group, and $a, b \in G$. Prove that (i) $(a^{-1})^{-1} = a$; and, $(ab)^{-1} = b^{-1}a^{-1}$. (K₃)

OR Let $G = \{1, -1, i, -i\}$, where $i := \sqrt{-1}$. Show that G is a cyclic group with respect to multiplication of complex numbers. (K₃)

(c) What is meant by a ring. Give an example of a commutative ring, and also of a non-commutative ring. (K₂)

Q 9 (CO3) Attempt any TWO questions. Each question is of 5 marks.

✓(a) Draw the Hasse diagram of the poset $A = \{3, 4, 6, 12, 24, 36, 48\}$ partially ordered by the relation divides. Also, find the minimal and maximal elements of this poset. (K₃)

✓(b) Show that the divisor lattice $(D_{48}, |)$ is not complemented. (K₃)

(c) Apply K-maps to find the minimal SOP for the Boolean expression (K₃)

$$F(u, v, w) = uv'w' + uvw' + uvw + u'v'w.$$

Also draw the logic circuits for the minimal SOP so obtained.

Q 10 (CO4) Attempt any TWO questions. Each question is of 5 marks.

✓(a) Check whether $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is tautology or a contradiction.. (K₃)

(b) Prove the validity of the argument "If the races are fixed or the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police will be happy. The police force is never happy. Therefore, the races are not fixed". (K₃)

(c) Transform the following sentences into logical expression by using connectives, predicates, and quantifiers: (i) Nobody loves everybody; (ii) No one has done every problem given in the assignment; and, (iii) Everyone has exactly one best friend. (K₃)

Q 11 (CO5) Attempt any TWO questions. Each question is of 5 marks.

(a) State handshaking theorem for a simple graph $G = (V, E)$, and hence deduce that the number of odd degree vertices in V is even. (K₃)

✓(b) Suppose a binary tree T has two traversals given by

in-order: BEHFACDGI and post-order: HFEABIGDC.

Construct T , and also determine the pre-order traversal of T . (K₃)

✓(c) Let a_n denote the number of valid n -digit codewords in decimal notation. It is known that a_n satisfy the recurrence relation

$$a_n = 8a_{n-1} + 10^{n-1}, \quad \text{with } a_1 = 9.$$

Use generating function method to find an explicit formula for a_n .

(K₃)