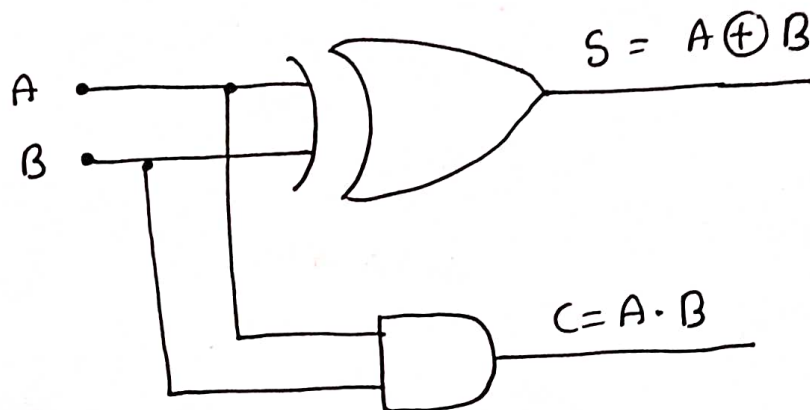


UNIT - 2

ARITHMETIC AND LOGIC UNIT

HALF ADDER:- A combinational circuit that performs the addition of two bits is called a Half Adder.

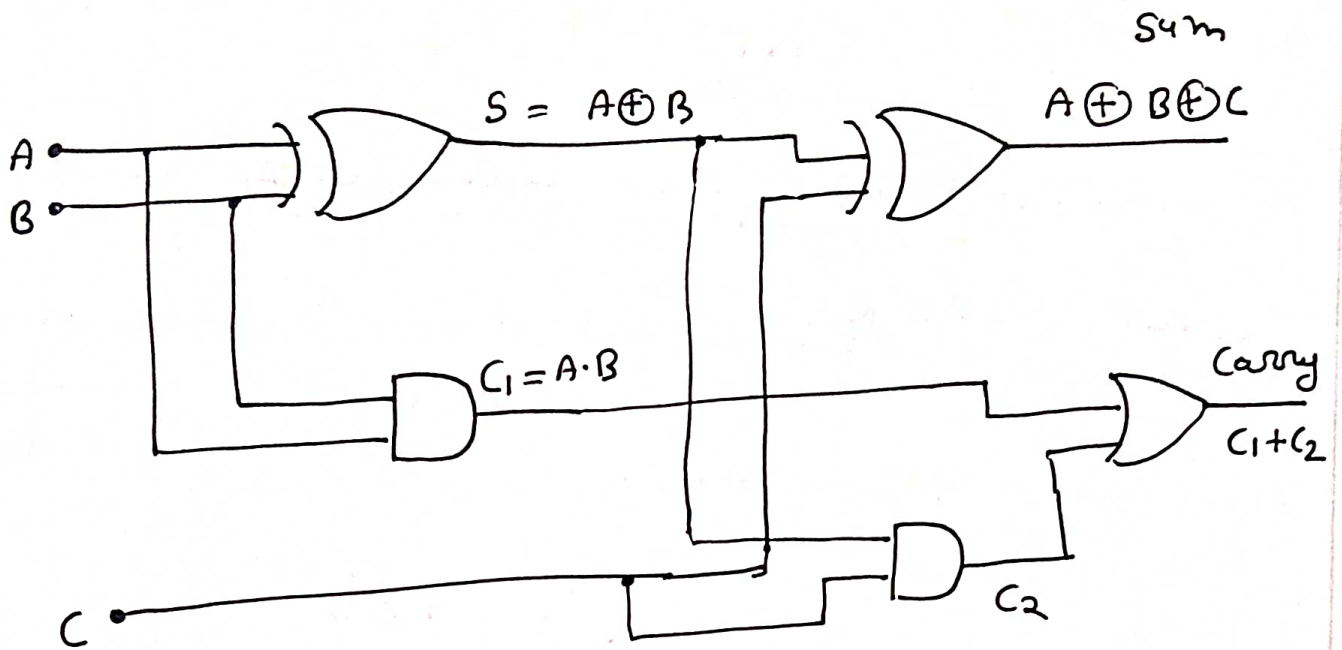


Half Adder truth table:-

A	B	$S = (A \oplus B)$ sum	$C = A \cdot B$ carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



FULL ADDER:- A combinational circuit that performs the addition of three bits, two significant bits and a carry bit, is called a full Adder.



Truth table of full Adder:-

Input			Output	
A	B	C	Sum-S	Carry-C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



PARALLEL BINARY ADDER:-

In the most of Logic circuit addition of more than 1-bit is carried out. The Addition of multibit numbers can be performed using several full adder.

The sum of 4-bit number can be done using 4-full Adders as shown in fig

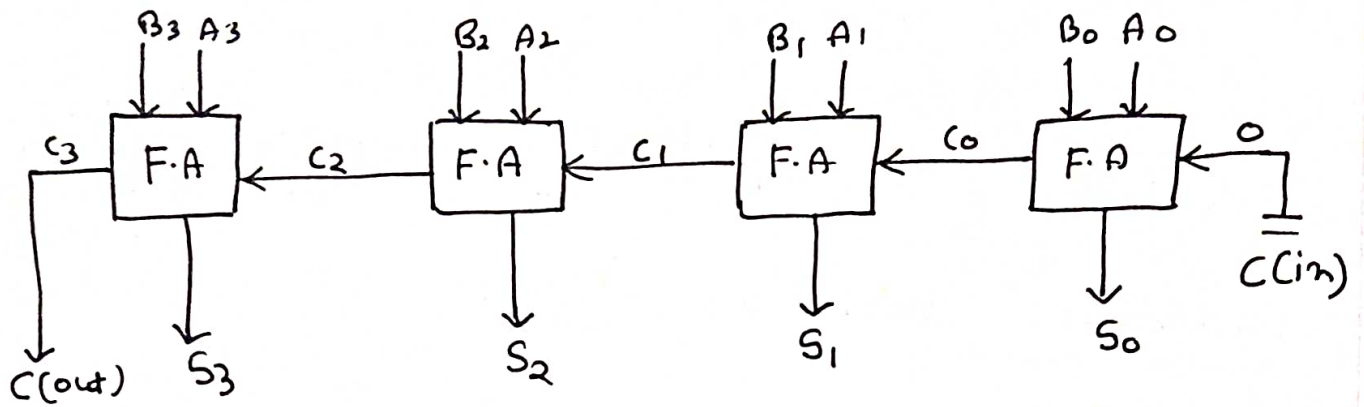
Let two 4-bit numbers A & B

$$A = 1111$$

$$B = 0011$$

$$\begin{array}{rcccc}
 A_3 & A_2 & A_1 & A_0 & = & 1 & 1 & 1 & 1 \\
 B_3 & B_2 & B_1 & B_0 & & + & 0 & 0 & 1 & 1 \\
 \hline
 & & & & & (1) & 0 & 0 & 1 & 0
 \end{array}$$

4-Bit Binary Parallel Adder (Ripple carry Adder):-



⇒ Since all the bits of the two numbers are fed into the adder circuit simultaneously and the addition in each position is taking place at the same time therefore this circuit is known as Parallel Adder.

⇒ This type of Adder is called ripple carry Adder. Because the output carry of lower stage is connected to the input carry of the next Higher stage.

⇒ Its speed of operation is limited by the carry Propagation delay through all stage.

⇒ To overcome carry Propagation delay a new adder is used which is known as Carry look ahead (CLA) Adder.

$G_i \Rightarrow$ Carry generate $\Rightarrow$ It produces carry

$P_i \Rightarrow$ Carry Propagate $\Rightarrow$ Propagation of the carry from C_i to C_{i+1}

The carry output of each stage:

$$\boxed{C_{i+1} = G_i + P_i \cdot C_i} \longrightarrow \textcircled{1}$$

for $i = 0$

$$C_1 = G_0 + P_0 \cdot C_0$$

$$\boxed{C_1 = A_0 B_0 + (A_0 \oplus B_0) \cdot C_0} \quad \text{All known terms}$$

for $i = 1$

$$C_2 = G_1 + P_1 \cdot C_1$$

$$C_2 = G_1 + P_1 (G_0 + P_0 \cdot C_0)$$

$$C_2 = G_1 + P_1 G_0 + P_1 P_0 \cdot C_0$$

$$\boxed{C_2 = (A_1 \cdot B_1) + (A_1 \oplus B_1) \cdot (A_0 B_0 + A_0 \oplus B_0) \cdot C_0}$$

for $i = 2$

$$C_3 = G_2 + P_2 \cdot C_2$$

$$C_3 = (A_2 \cdot B_2) + (A_2 \oplus B_2) \cdot C_2$$

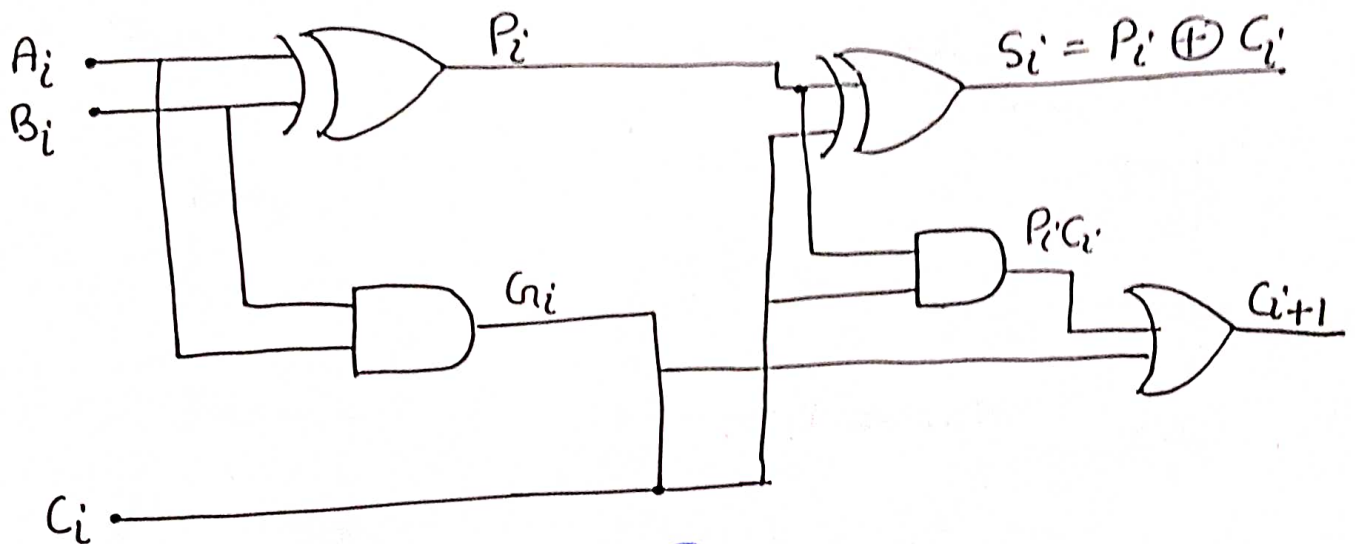
M.S

(2)

Carry Look Ahead Adder: It is based on the principle of looking at the lower order bits of the inputs if a higher order carry is generated.

⇒ two functions:

- (1) Carry Propagation (P_i)
- (2) Carry generation (G_i)



$$P_i = A_i \oplus B_i$$

$$G_i = A_i \cdot B_i$$

Sum $S_i = P_i \oplus C_i$

Carry $C_{i+1} = G_i + P_i C_i$

where :-

M.S

$$C_3 = G_2 + P_2 (G_1 + P_1 (G_0 + P_0 (C_0)))$$

$$C_3 = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0$$

C_3 does not wait for C_2 & C_1 to propagate.

C_3 is estimated at the same time as C_2 & C_3 are estimated.

The sum output of each stage:

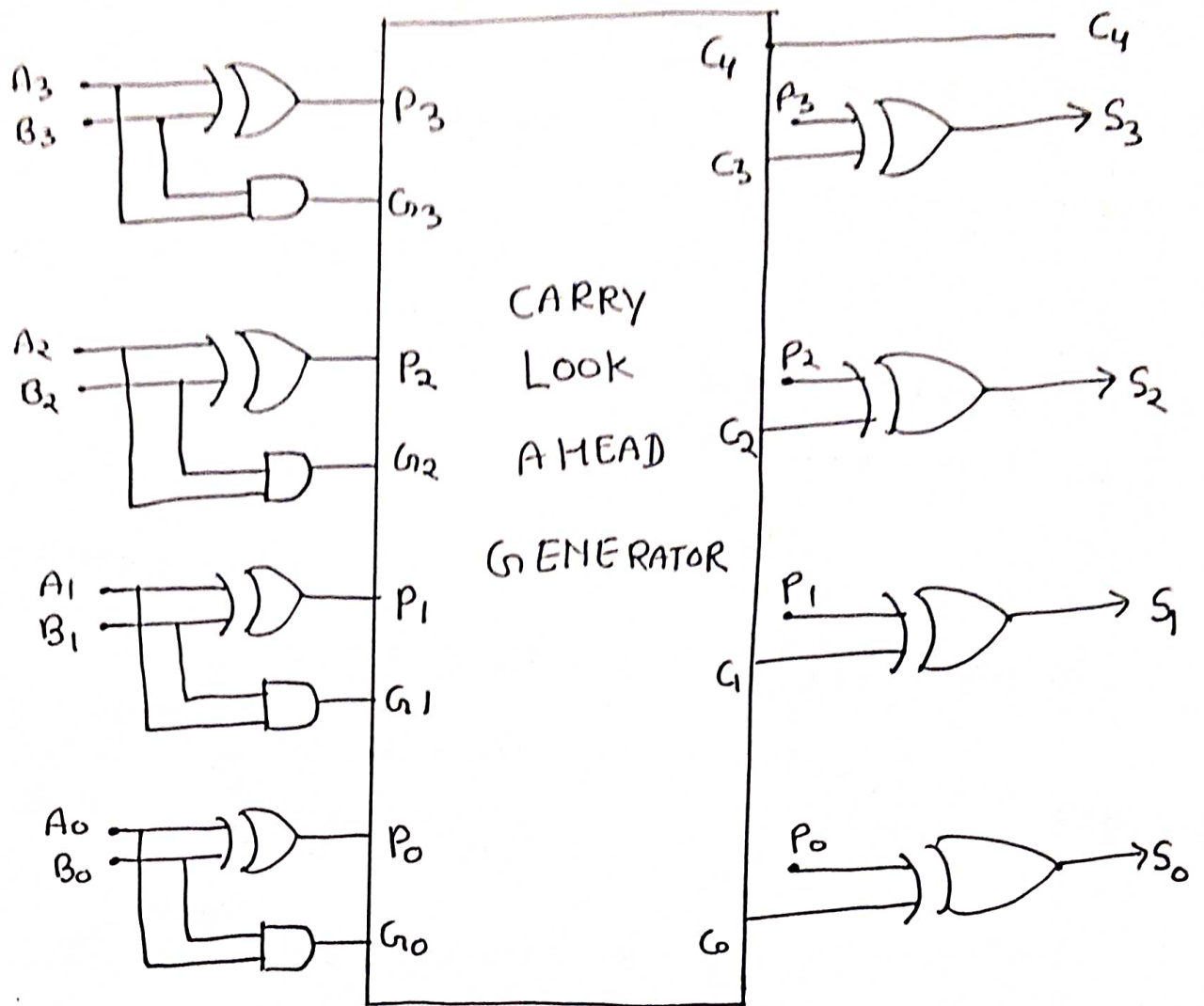
$$S_0 = P_0 \oplus C_0 \Rightarrow A_0 \oplus B_0 \oplus C_0$$

$$S_1 = P_1 \oplus C_1 \Rightarrow A_1 \oplus B_1 \oplus C_1$$

$$S_2 = P_2 \oplus C_2 \Rightarrow A_2 \oplus B_2 \oplus C_2$$

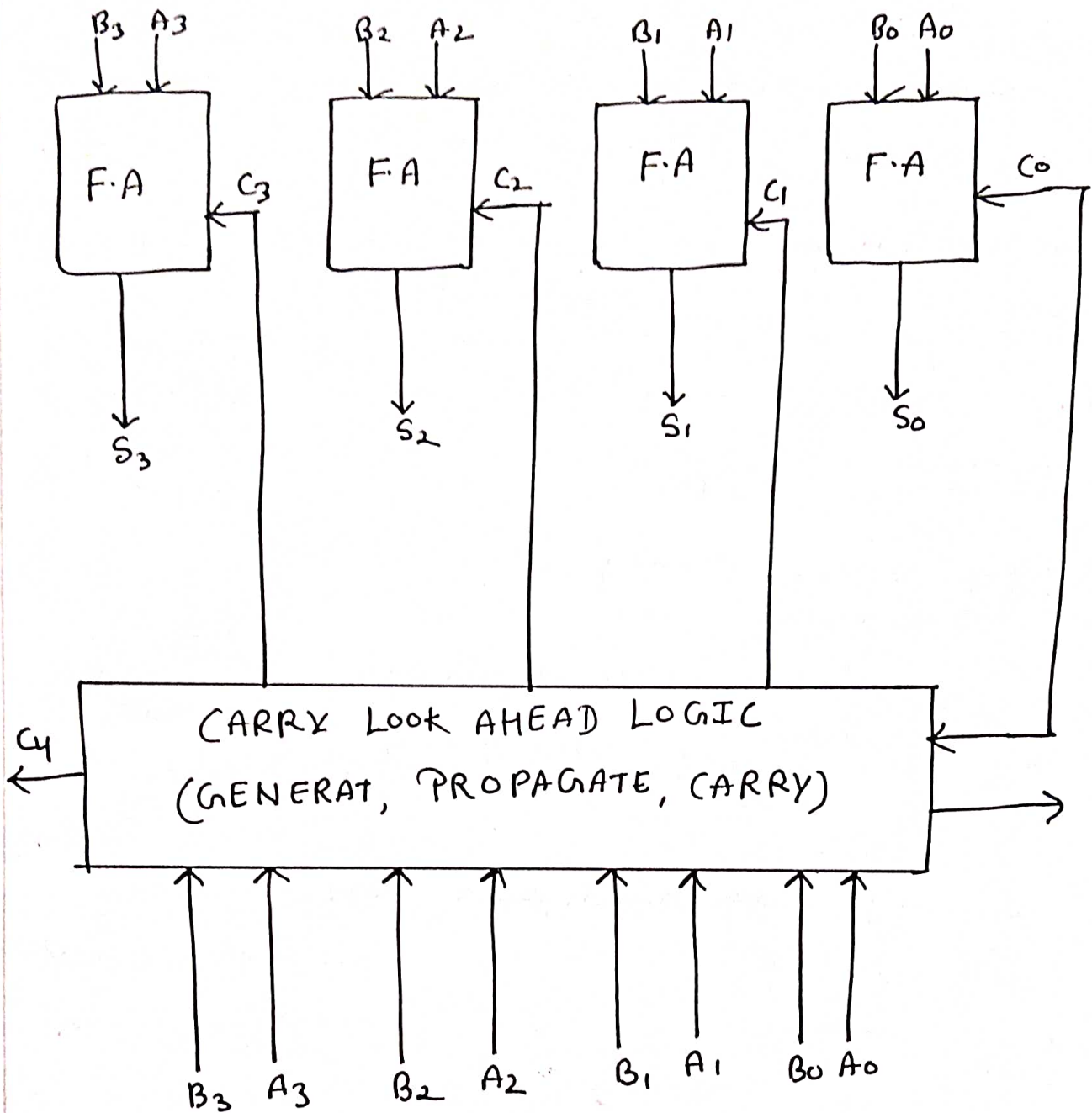
$$S_3 = P_3 \oplus C_3 \Rightarrow A_3 \oplus B_3 \oplus C_3$$

Logic Diagram Carry Look Ahead Adder:-



(Logic Diagram carry lookahead Adder)

Special Look Ahead carry generator CKT:-



(Special Look AHEAD Carry generator CKT)

ARRAY MULTIPLIER:- Array multiplier has regular structure. Multiplier circuit is based on add and shift algorithm. Each Partial Product is generated by multiplication of

multiplier with one multiplier bit. The Partial Product are shifted according to their bit orders and then added.

The addition can be performed with normal carry propagate adder.

$N-1$ adders are required where N is multiplier length.

4×4 Array multiplier:-

$x \times y$ $x \Rightarrow$ multiplicand
 $y \Rightarrow$ multiplier

$4 \times 4 \Rightarrow$ Array multiplier $\Rightarrow$ Hardware circuit for fast multiplication

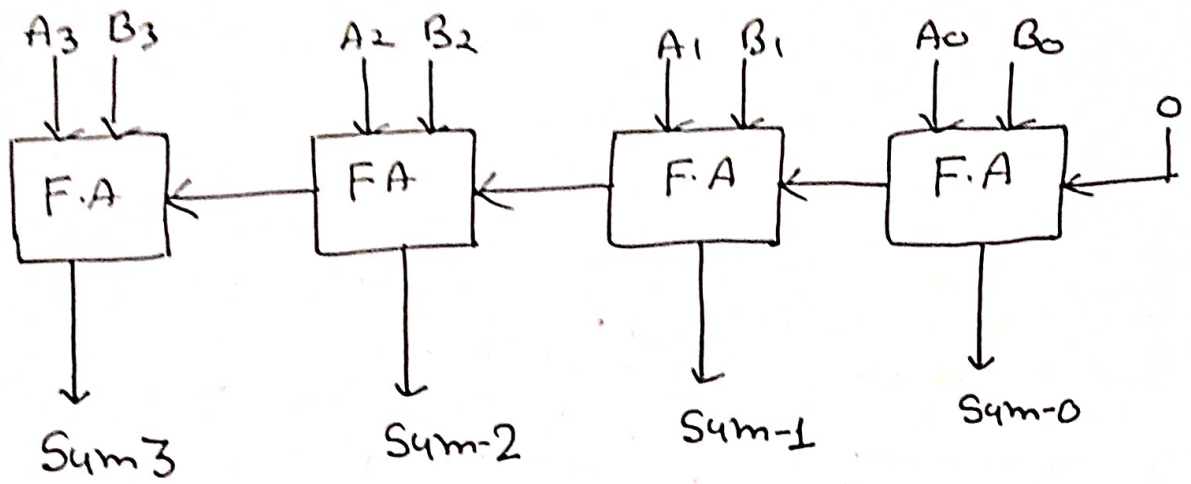
x y
 $\downarrow$ $\downarrow$
 n -bit n -bit

(full adder / Half adder / AND gate)

Carry Propagate Adder:-

connecting full-adders to make a multi-bit carry propagate adder.

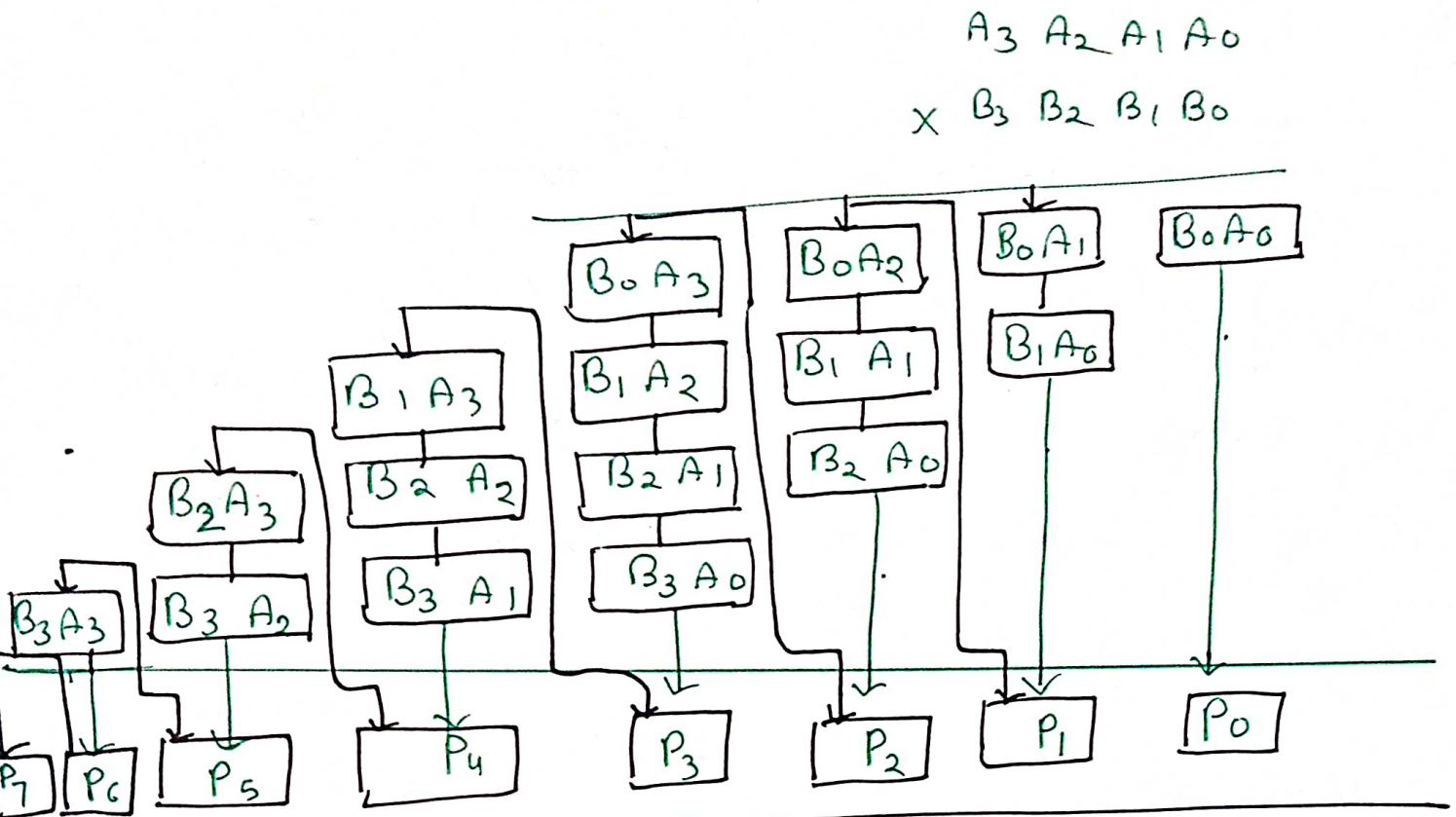
M.S



$$x = x_3 x_2 x_1 x_0 \quad y = y_3 y_2 y_1 y_0$$

$$x \times y = \begin{array}{r} x_3 x_2 x_1 x_0 \\ \times y_3 y_2 y_1 y_0 \\ \hline \end{array}$$

$$\begin{array}{r} x_3 y_0 \quad x_2 y_0 \quad x_1 y_0 \quad x_0 y_0 \\ x_3 y_1 \quad x_2 y_1 \quad x_1 y_1 \quad x_0 y_1 \\ x_3 y_2 \quad x_2 y_2 \quad x_1 y_2 \quad x_0 y_2 \\ x_3 y_3 \quad x_2 y_3 \quad x_1 y_3 \quad x_0 y_3 \\ \hline c \quad s_6 \quad s_5 \quad s_4 \quad s_3 \quad s_2 \quad s_1 \quad s_0 \end{array}$$



Hardware Requirement

16 - AND Gate

4 - Half Adder

8 - Full Adder

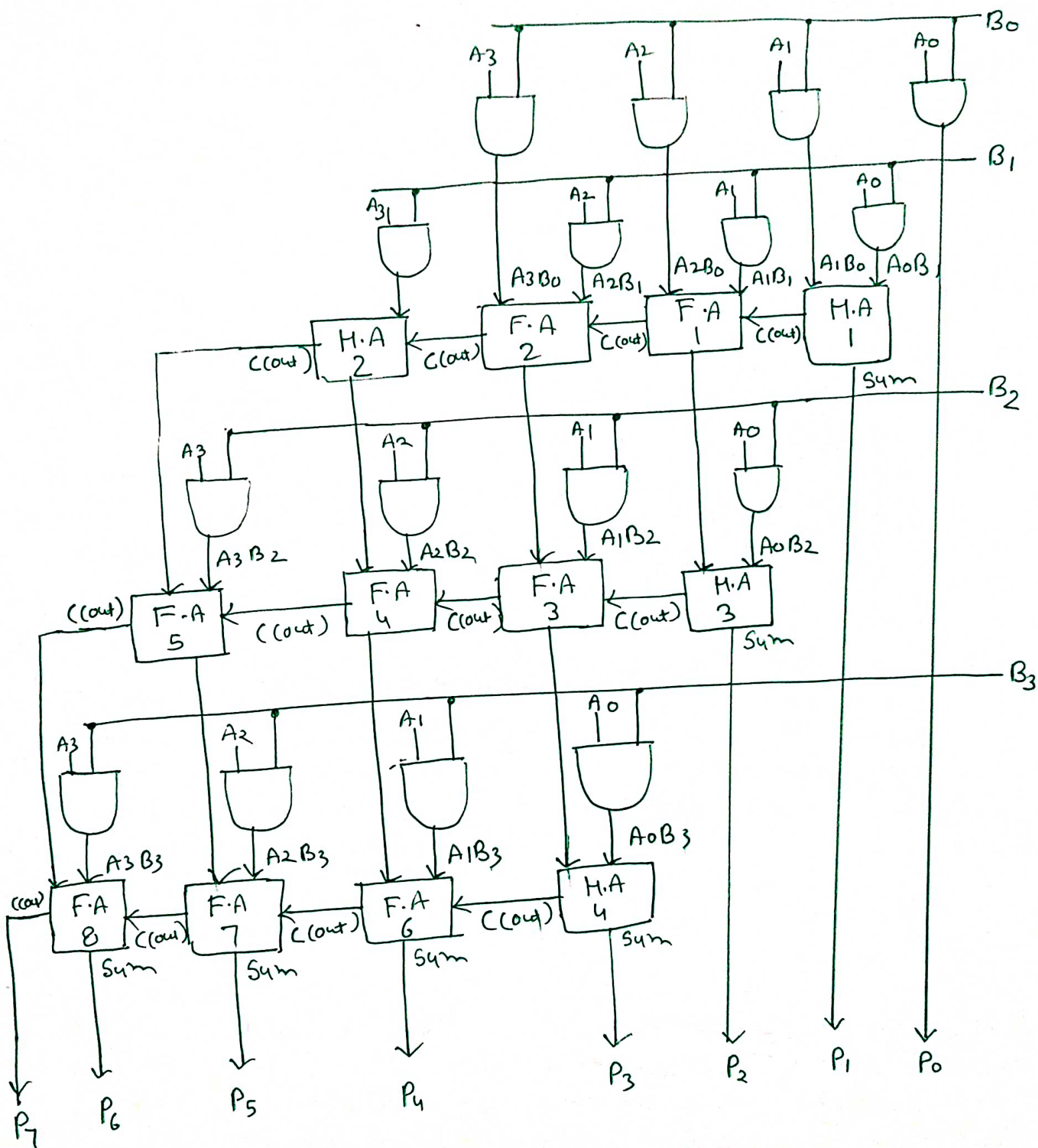
total 12 - Adders

$$\begin{aligned}
 m \times n &= m \Rightarrow 4 \text{ (No of Bits in A)} \\
 & \quad n \Rightarrow 4 \text{ (No of Bits in B)}
 \end{aligned}$$

$$m \times n = \text{AND gates}$$

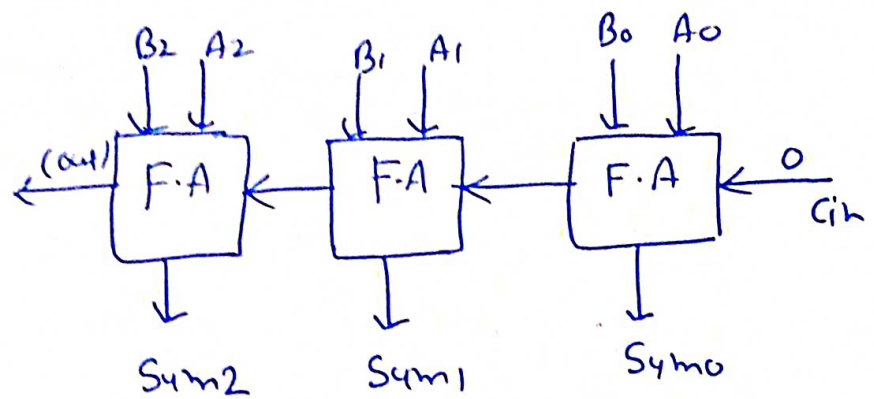
$$n = \text{No of H.A (Half Adder)}$$

$$n(n-2) = \text{No of F.A (Full Adder)}$$



(4x4 Array multiplier)

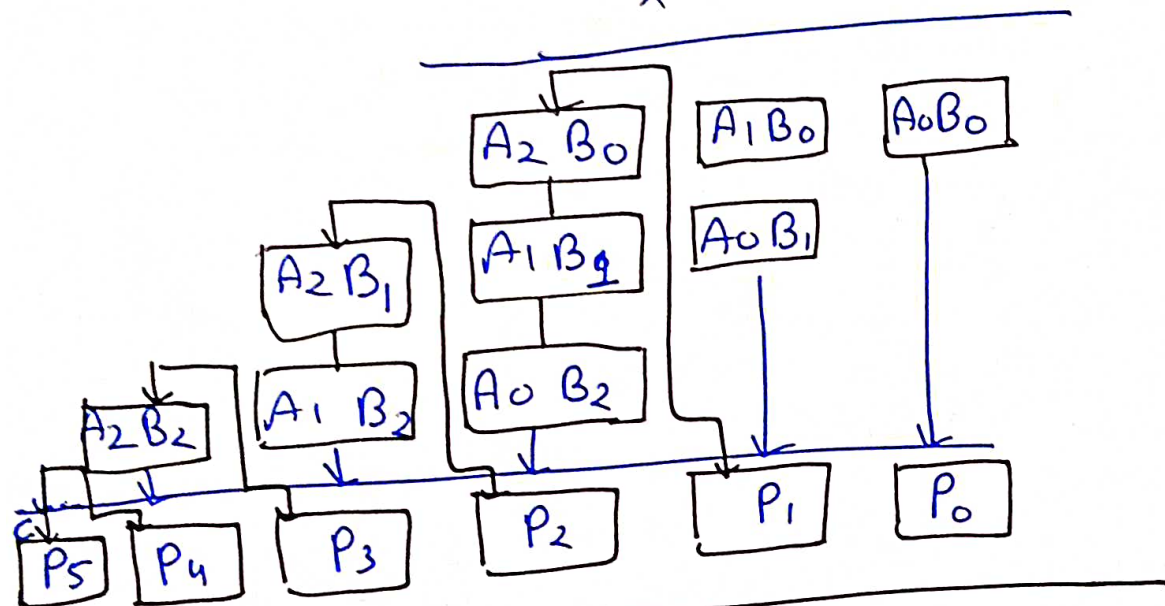
3-bit Binary multiplier:-



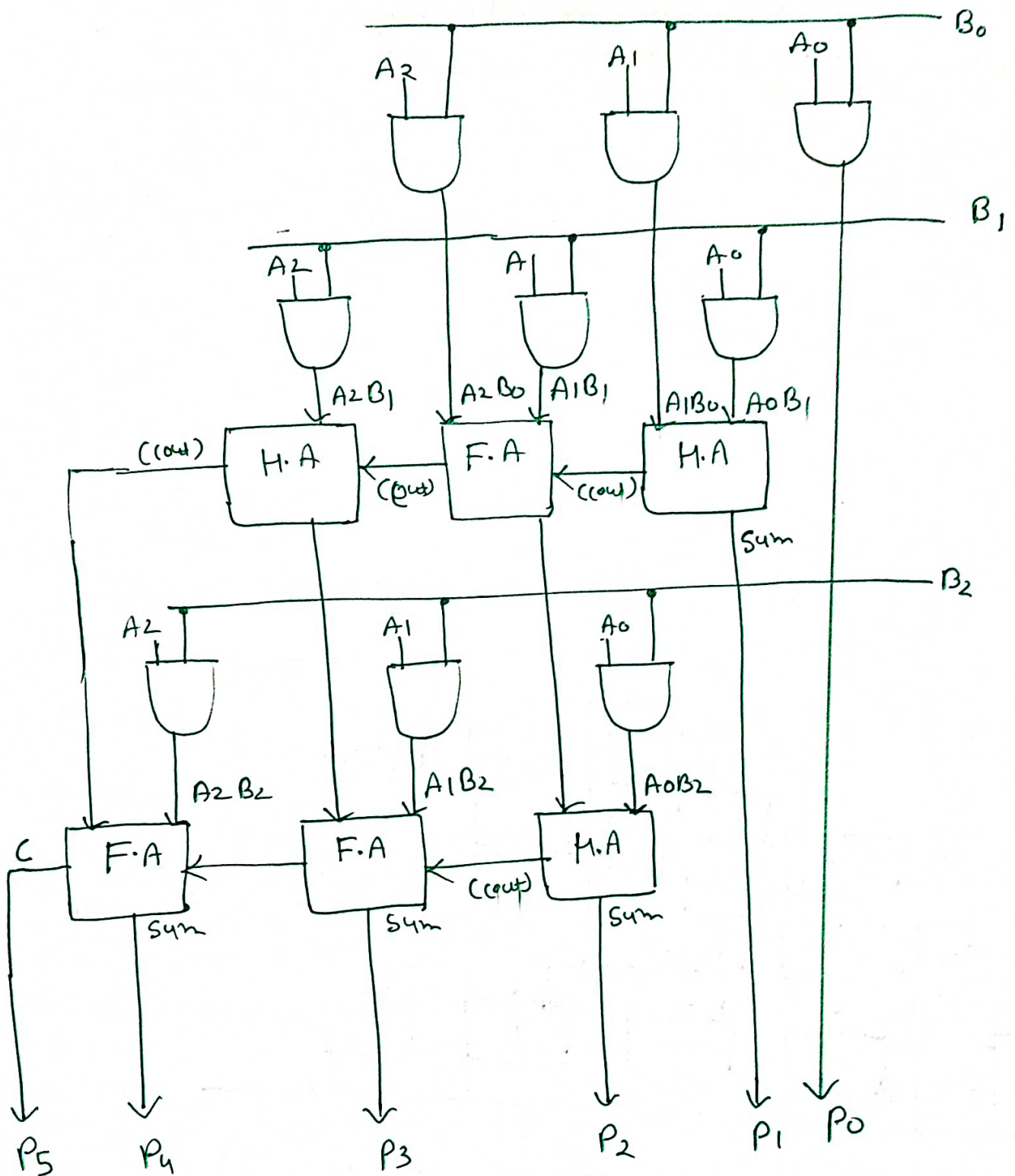
$$X = \cancel{x_2}$$

$$A = A_2 A_1 A_0 \quad B = B_2 B_1 B_0$$

$$A \times B = \begin{array}{r} A_2 A_1 A_0 \\ \times B_2 B_1 B_0 \\ \hline \end{array}$$



(3x3) Array multiplier:-



Handware Requirement

N-bit multiplication

AND Gates $\Rightarrow N^2 = 9$

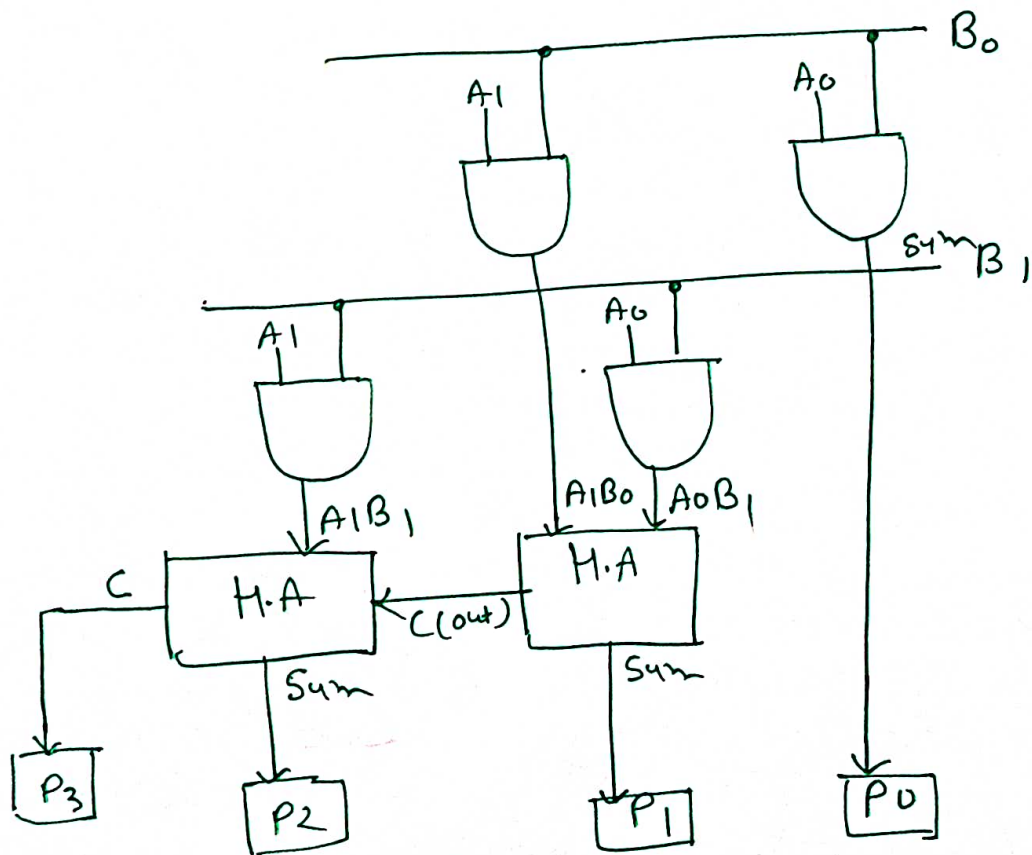
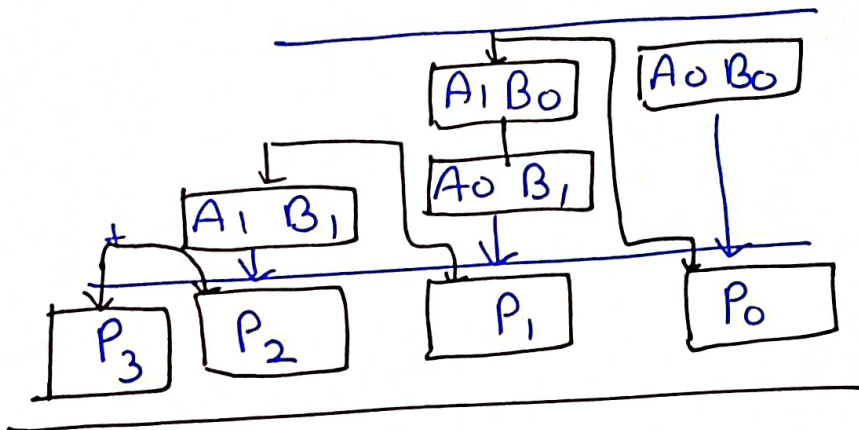
Half Adder $\Rightarrow N = 3$

Full Adder $= N(N-2) = 3$

2-bit Binary Array multiplier;

$$A = A_1 A_0 \quad B = B_1 B_0$$

$$A \times B = \begin{array}{r} A_1 A_0 \\ \times B_1 B_0 \\ \hline \end{array}$$



(2x2 Array multiplier)

BOOTH'S ALGORITHM

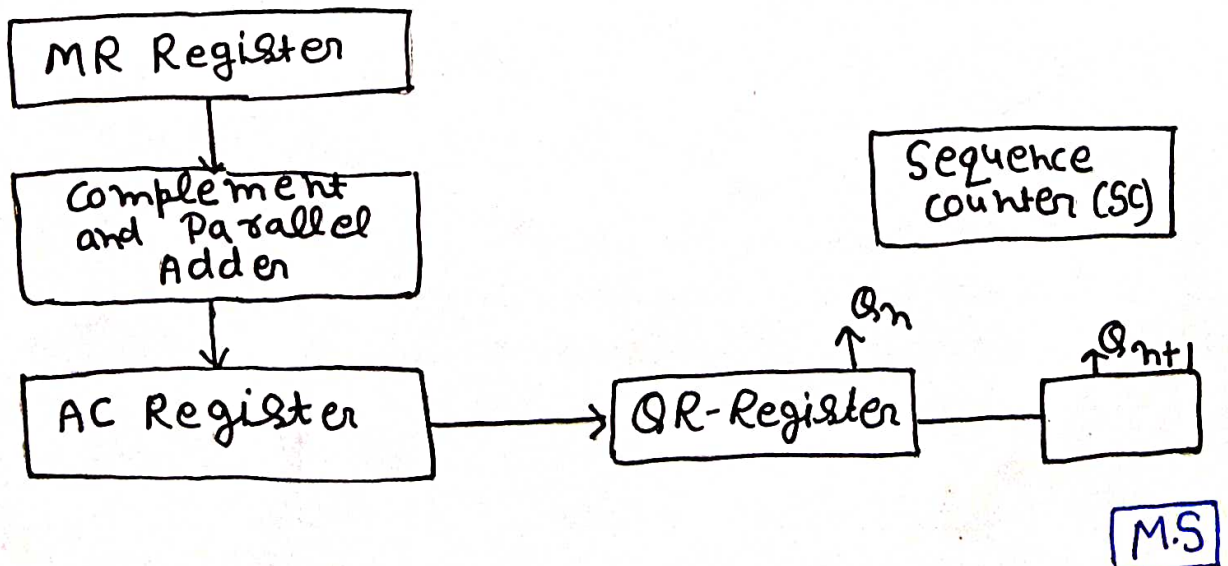
Booth's multiplication Algorithm:-

multiplication of signed 2's complement integers:-

Shifting and addition/subtraction rules for multiplicand in Booth's Algorithm.

- ① The multiplicand is subtracted from the Partial Product upon encountering the first least significant 1 in a string of 1's in a multiplier.
- ② The multiplicand is added to the Partial Product upon encountering the first 0 (provided that there was a previous 1) in a string of 1's in the multiplier.
- ③ The Partial Product does not change when the multiplier bit is identical to the previous multiplier bit.

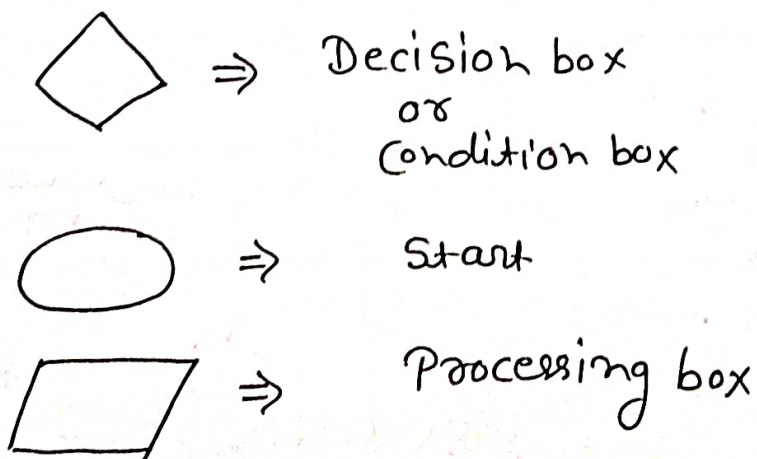
Hardware Implementation for Booth Algorithm:-

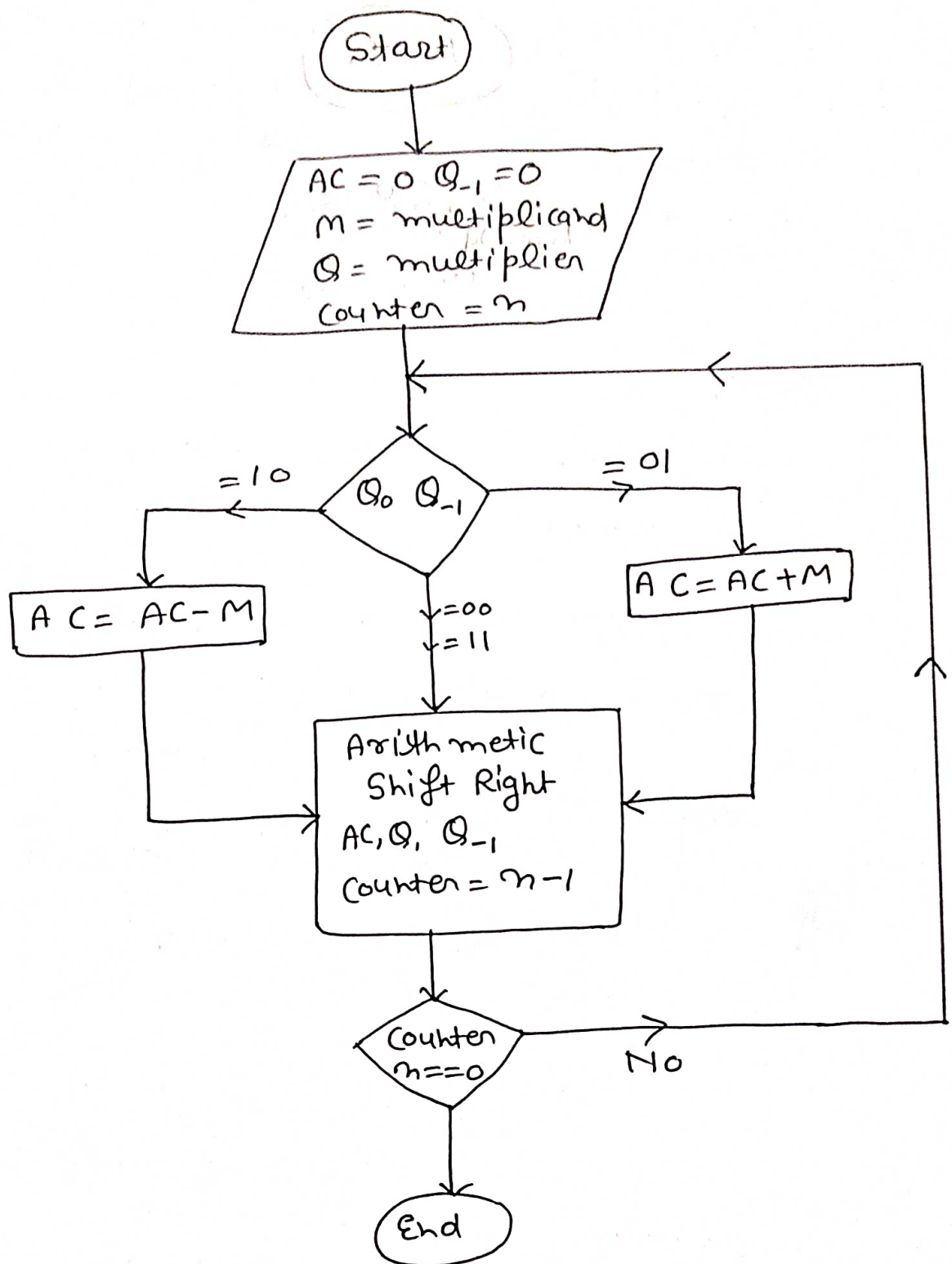


- ⇒ Here sign is not separated from register.
- ⇒ QR Register contains the multiplier registers and Q.
- ⇒ Q_n represent LSB of the multiplier in QR.
- ⇒ Q_{n+1} is an extra flip-flop appended to QR to facilitate a double bit inspection of the multiplier.
- ⇒ AC Register and Q_{n+1} are initially cleared to 0
- ⇒ SC is a set to the number of bits in the multiplier.
- ⇒ Q_n Q_{n+1} are two successive bits in the multiplier.

FLOW CHART for multiplication of signed 2's complement numbers

FLOW CHART FOR BOOTH'S MULTIPLICATION:-





For signed number multiplications three cases are possible.

- (1) + +
- (2) + -
- (3) - -

NOTE:- if n (Register-bit) value is not given
 $M+1$ Add one bit in multiplicand other-
 wise n value is depend given the Number
 of bit in the Questions.

Q₁ $+7 \times +3$ Using Booths multiplications.

Sol $M = 7$ $Q = 3$

$$M = 0111$$

$n = \text{number of bit in } M+1$

$$Q = 0011$$

$$1's \text{ complement of } M = 1000$$

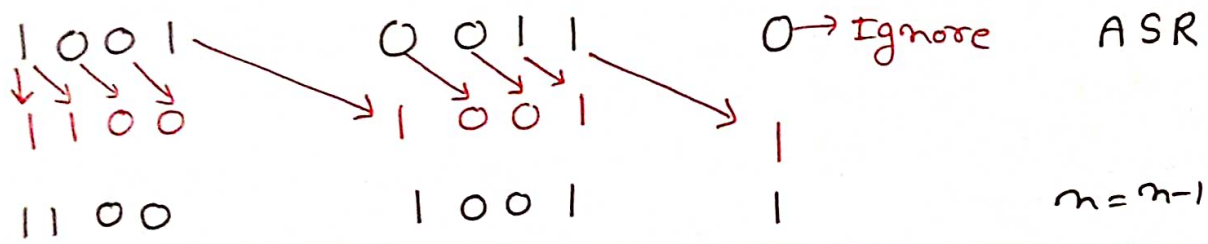
$$2's \text{ complement of } (-M) = + \frac{1}{1001}$$

$$(-M) = (1001)_2$$

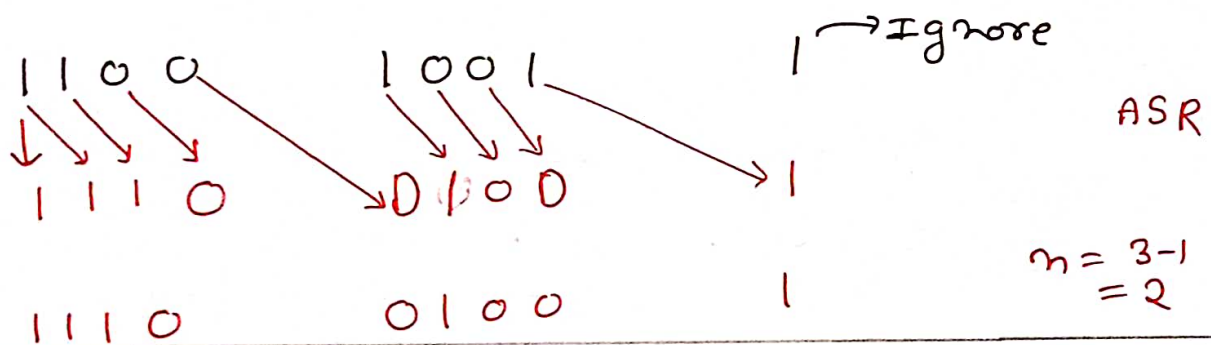
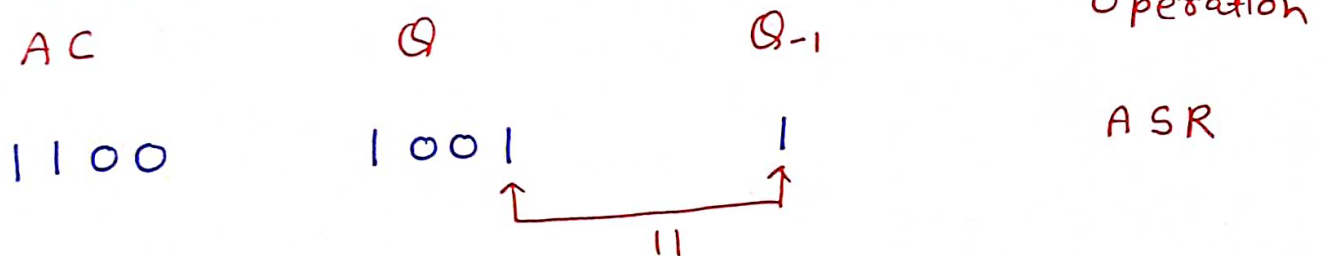
$$AC = 0000 \quad Q_{-1} = 0 \quad n = 4$$

Step-1:-

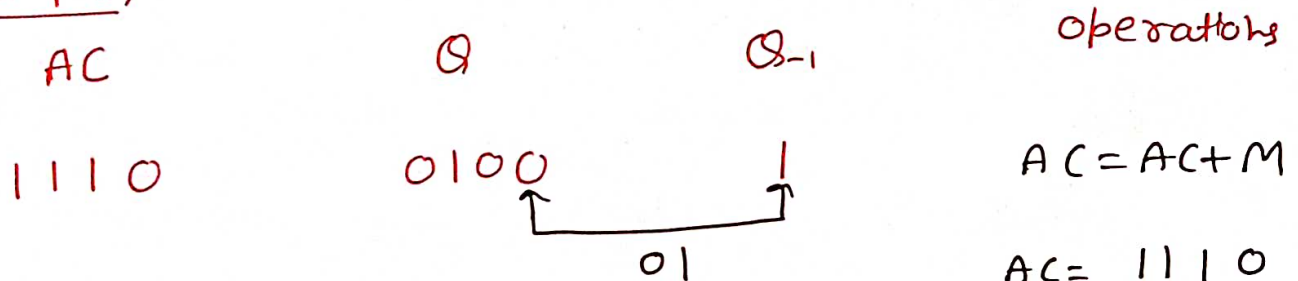
AC	Q	Q ₋₁	operation
0000	Q_3, Q_2, Q_1, Q_0 0011	0	Initially
0000	0011	0	$AC = AC - M$ $AC = \begin{array}{r} 0000 \\ 1001 \\ \hline 1001 \end{array}$



Step-2:

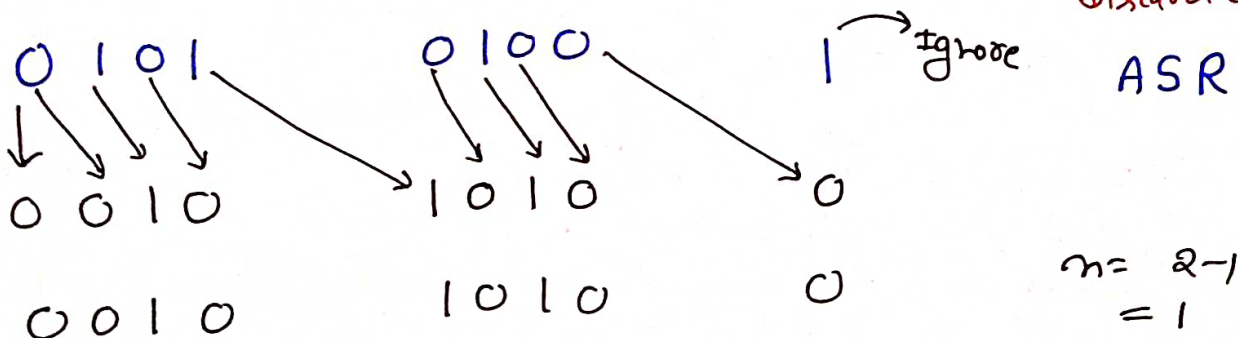


Step 3:



$$AC = \begin{array}{r} 1110 \\ 0111 \\ \hline \end{array}$$

$\textcircled{1}0101$
 ↓
 Discard carry



M.S

Step-4'
AC

Q

Q₁

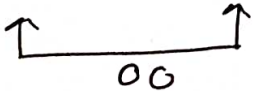
operation

0010

1010

0

ASR

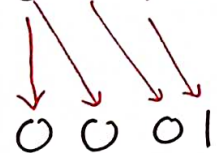


0010

1010

0 → ignore

ASR



0001

0101

0

n = 1-1
= 0

⇒ AC, Q

⇒ 0001 0101



00010101

⇒ (21)₁₀

(+7) × (+3) = (21)₁₀ Ans

Q₂ (-7) × (+3) using Booth's multiplication
n = 5 bit.

Sol M = -7 Q = 3

M = (-7) = 00111

Q = (3) = 00011

-M = 00111

M.S

1's complement of $M = 11000$

2's complement of $M = + \frac{1}{11001}$

$$-(-M) = 11001$$

$$M = (11001)_2$$

$$Q = (00011)_2$$

$$AC = 00000 \quad Q_{-1} = 0$$

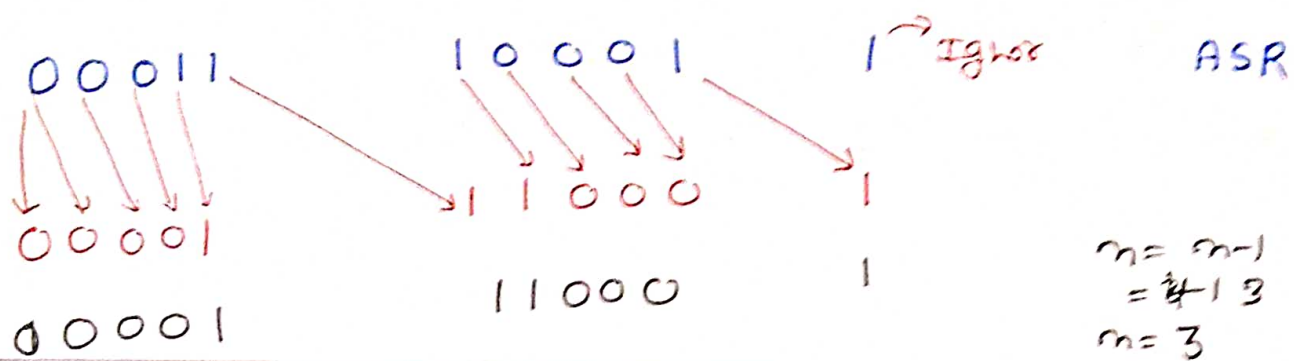
Step 1:-

AC	Q	Q ₋₁	operation
00000	^{Q₄Q₃Q₂Q₁Q₀} 00011	0	Initially
		↑ 10	
00000	00011	0	AC = A + (-M)
			$ \begin{array}{r} AC = 00000 \\ 00111 \\ \hline 00111 \\ \hline \text{ASR} \end{array} $
00111	00011	0 → Ignore	ASR
00011	10001	1	n = n - 1 = 5 - 1 n = 4

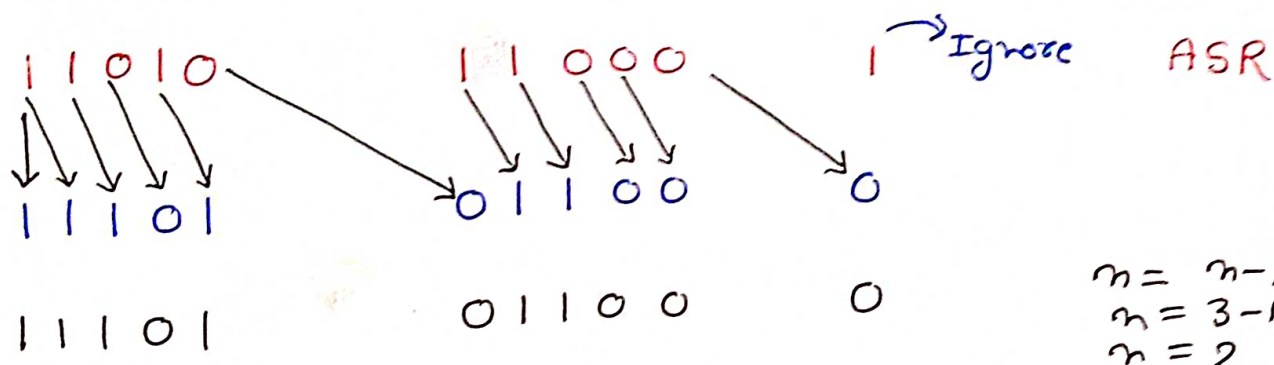
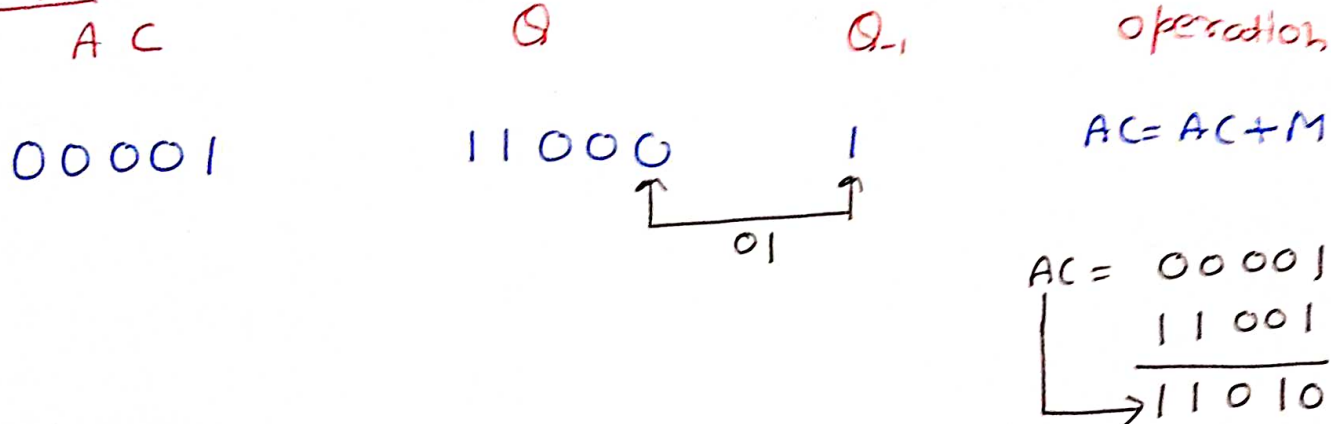
Step 2:-

AC	Q	Q ₋₁	operation
00011	10001	1	ASR
		↑ 11	

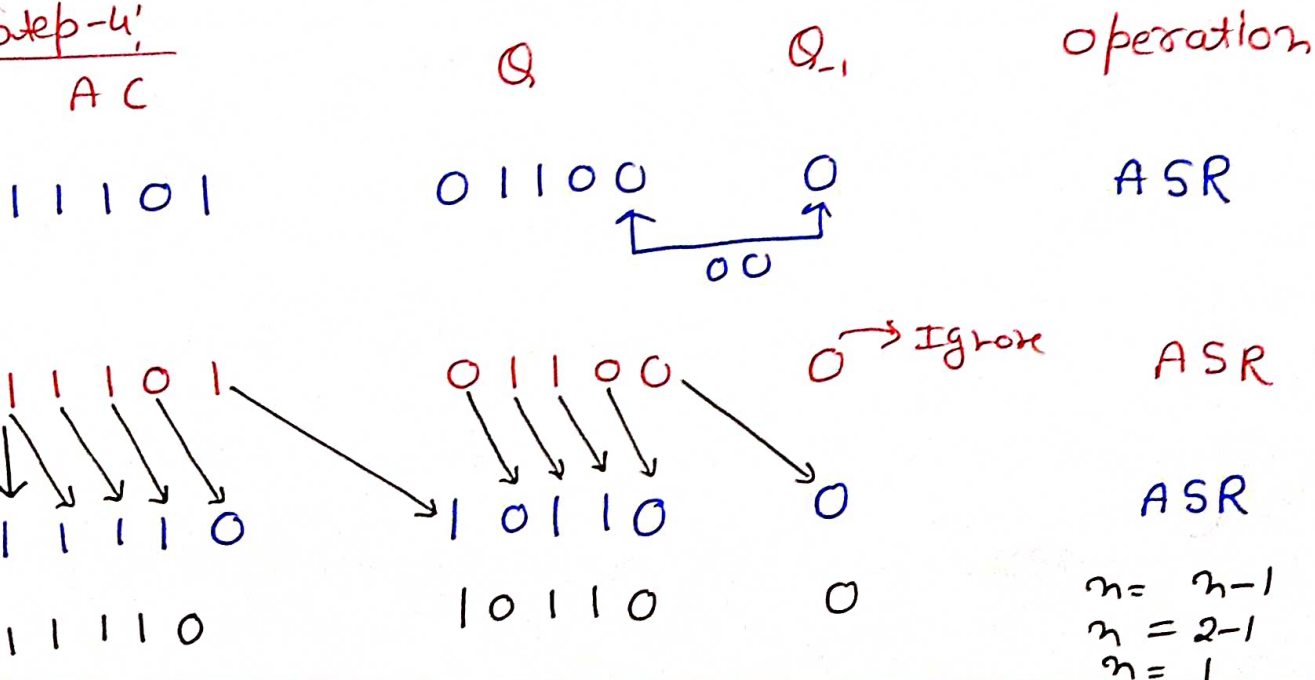
M.S



Step-3



Step-4'



M.S

Step-5

AC	Q	Q ₋₁	operation
11110	10110	0	ASR
		00	
11110	10110	0	ASR
11111	01011	0	Ignore
11111	01011	0	ASR
			$n = n-1$ $n = 1-1$ $n = 0$

$\Rightarrow$ AC, Q

11111 01011
 $\swarrow \searrow$ merge

$P \Rightarrow (1111101011)_2$

1's complement of $P = 0000010100$

2's complement of $P = + \frac{1}{0000010101}$

$-P = (10101)_2$

$-P = 21$

$(-7) \times (+3) = (-21)_{10} = (10101)_{10}$

Q3 -7×-3 using Booth's multiplication. Register bit is 4.

$$\Rightarrow (-ve) \times (-ve) \Rightarrow +(\text{Product})$$

Sol

$$M = (-7)_{10} \Rightarrow (0111)_2$$

$$1's \text{ complement of } M = 1000$$

$$2's \text{ complement of } (-M) = \frac{1}{1001}$$

$$(-M) = (1001)_2$$

$$M = (-7)_{10} = (1001)_2$$

$$-M = (7)_{10} = (0111)_2$$

$$Q = (-3)_{10} = (0011)_2$$

$$1's \text{ complement of } Q = 1100$$

$$2's \text{ complement of } (-Q) = \frac{1}{1101}$$

$$Q \Rightarrow (-3)_{10} = (1101)$$

$$AC = 0000 \quad Q_{-1} = 0 \quad -M = (0111)_2$$

$$Q = 1101$$

Step 1:

AC	Q	Q ₋₁	operation
0000	Q ₃ Q ₂ Q ₁ Q ₀ 1101	0	Initially
0000	1101	0	AC = AC - M
			M.S

↑ 10

0111
↓ ↓ ↓ ↓
0011

0011

1101
↓ ↓ ↓ ↓
1110

1110

0 → Ignore

AC = 0000
0111
—
0111

ASR

$n = n - 1 = 4 - 1$
 $n = 3$

Step-2:

AC

0011

Q

1110

Q₋₁

1

01

Operation

AC = AC + M

AC = 0011

1001
—
1101

1101
↓ ↓ ↓ ↓
1110

1110

1110
↓ ↓ ↓ ↓
1111

1111

1

→ Ignore ASR

ASR

$n = n - 1 = 3 - 1$
 $n = 2$

Step-3:

AC

1110

Q

0111

Q₋₁

0

10

Operation

AC = AC - M

AC = 1110
0111
—
① 0101

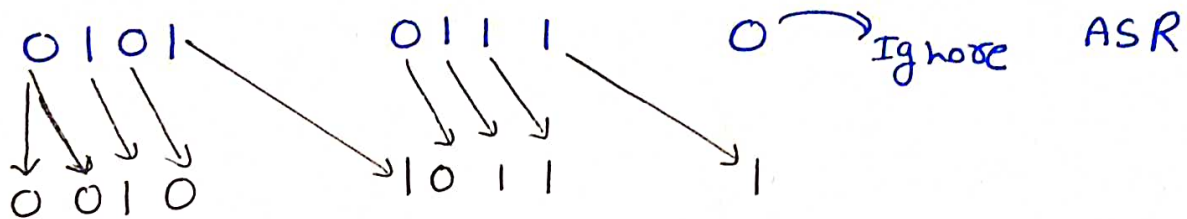
↓
Discard carry

M.S

0101

0111

0



0010 1011 1

$n = n-1 = 2-1 = 1$

Step 4:

AC

Q

Q₁

operation

0010

1011

1

ASR



0010

1011



Ignore ASR

0001

0101

1

ASR

0001

0101

1

$n = n-1 = 1-1 = 0$

$P \Rightarrow AC, Q$

$\Rightarrow 0001 \quad 0101$



00010101

$\Rightarrow (21)_{10}$

$(-7) \times (-3) = (21)_{10}$ Ans

M.S

Example Show the multiplication Process using Booth's algorithm when the following numbers are multiplied (-13) by (-8) .

sol $M = -13$ $Q = 8$

$-M = 01101$ $Q \Rightarrow 01000$

number of bit $n = 5$

1's complement of $M = 10010$

2's complement of $-M = \begin{array}{r} + \quad 1 \\ \hline 10011 \end{array}$

$-(-M) = 10011$

$M = 10011$

$-M = 01000$

$AC = 00000$

$Q_{-1} = 0$

$n = 5$

Step 1

AC	Q	Q_{-1}	operation
00000	01000	0	Initially
00000	01000	0	Ignore
00000	00100	0	ASR

M.S

00000

00100

0

$$n = n - 1 \\ = 5 - 1 \\ = 4$$

Step 2:

A C

Q

Q₋₁

Operation

00000

00100 0
00

ASR

00000
↓ ↓ ↓ ↓ ↓
00000

00100
↓ ↓ ↓ ↓ ↓
00010

0 → Ignore ASR

00000

00010

0

$$n = n - 1 = 4 - 1 \\ = 3$$

Step 3:

A C

Q

Q₋₁

Operation

00000

00010 0
00

ASR

00000
↓ ↓ ↓ ↓ ↓
00000

00010
↓ ↓ ↓ ↓ ↓
00001

0 → Ignore ASR

00000

00001

0

$$n = n - 1 = 3 - 1 \\ = 2$$

Step 4

A C

Q

Q₋₁

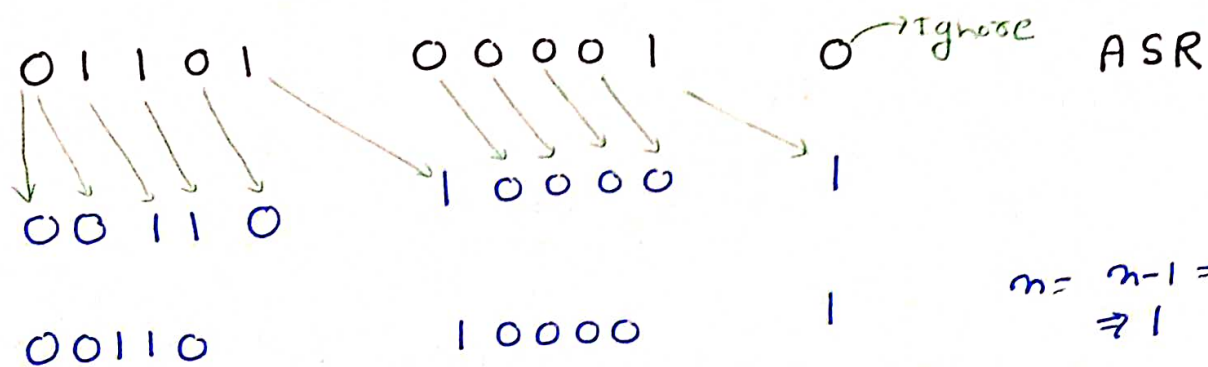
Operation

00000

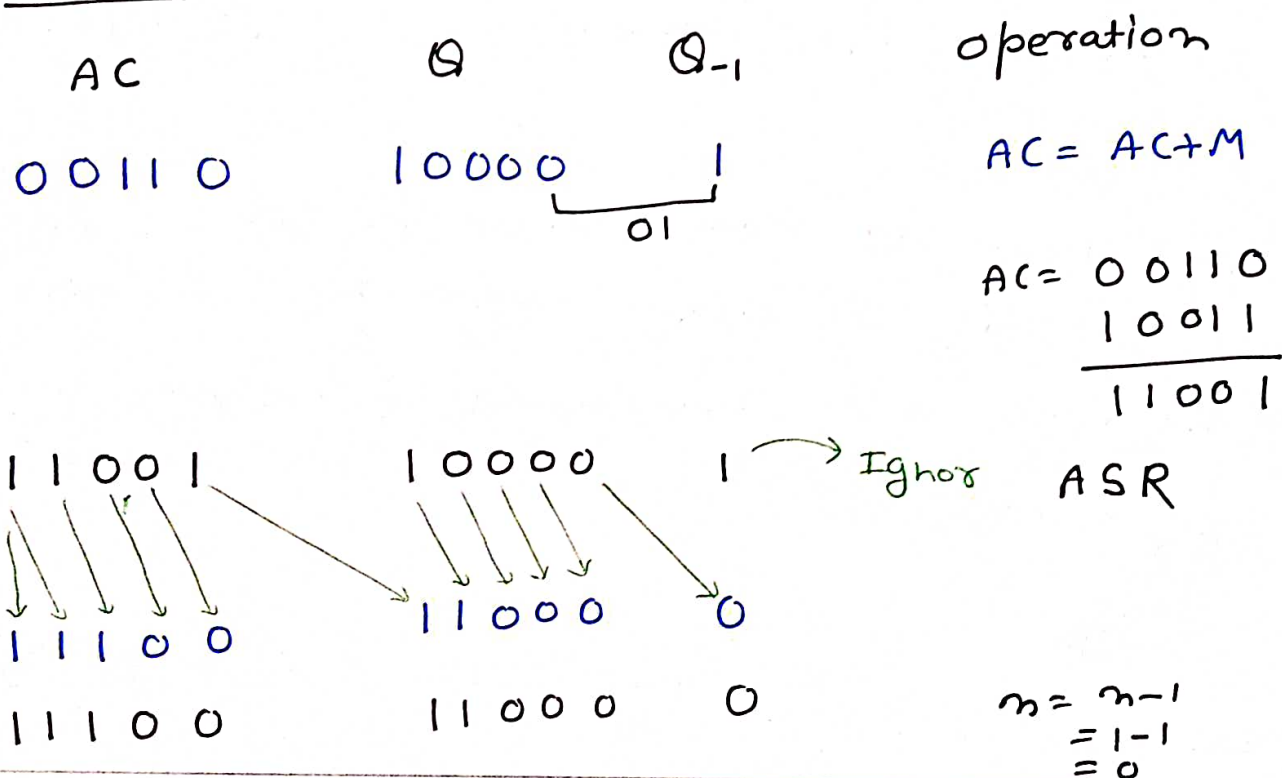
00001 0
10

AC = AC - M

$$AC = \begin{array}{r} 00000 \\ 01101 \\ \hline 01101 \end{array}$$



Step-5



Result = AC, Q .

$\Rightarrow 11100 \quad 11000$

merge

$R \Rightarrow (1110011000)_2$

1's complement of $R = 0001100111$

2's complement of $(R) = + \frac{1}{0001101000}$

M.S

$$(-R) \Rightarrow 1101000$$

$$R = -(104)_{10} \underline{Ans}$$

Ex Show the multiplication Process using Booth's algorithm when the following numbers are multiplied $(+13)$ by $(+8)$.

Ex Show the multiplication Process using Booth's algorithm when the following numbers are multiplied (-13) by (-8) .

Do your self

Example

Show the step by step the multiplication process using Booth's algorithm when $(+15)$ and (-13) numbers are multiplied. Assume 5-bit registers that hold signed numbers.

Sol

$$M = 15 \quad Q = -13 \quad n = 5$$

$$M \Rightarrow 01111 \quad Q = 01101$$

$$1's \text{ complement of } M = 10000$$

$$2's \text{ complement of } (-M) = \begin{array}{r} + \quad 1 \\ \hline 10001 \end{array}$$

$$-M = 10001$$

$$-Q = 01101$$

$$1's \text{ complement of } (-Q) = 10010$$

$$2's \text{ complement of } -(-Q) = \begin{array}{r} + \quad 1 \\ \hline 10011 \end{array}$$

$$Q = 10011$$

$$M = 01111 \quad Q = 10011 \quad -M = 10001$$

$$AC = 00000 \quad Q_1 = 0$$

Step-1

AC

Q

Q₁

operation

$$00000 \quad 10011 \quad \begin{array}{|c|} \hline 0 \\ \hline 10 \end{array}$$

Initially

$$AC = AC - M$$

(M.S)

$$AC = \begin{array}{r} 00000 \\ 10001 \\ \hline 10001 \end{array}$$

10001
↓ ↓ ↓ ↓ ↓
11000

11000

10011
↓ ↓ ↓ ↓ ↓
11001

11001

0 → Ignore

ASR

ASR

$$n = n - 1 \\ = 5 - 1 = 4$$

Step-2

AC

Q

Q₋₁

operation

11000
↓ ↓ ↓ ↓ ↓
11100

11100

11001
↓ ↓ ↓ ↓ ↓
01100

01100

1 → Ignore

ASR

ASR

$$n = n - 1 \\ = 4 - 1 \\ = 3$$

Step-3

AC

Q

Q₋₁

operation

11100

01100

1
01

AC = AC + M

$$AC = \begin{array}{r} 11100 \\ 01111 \\ \hline 00101 \end{array}$$

00101

Discard carry

ASR

ASR

01011
↓ ↓ ↓ ↓ ↓
00101

01100
↓ ↓ ↓ ↓ ↓
10110

1 → ignore

0

M.S

00101

10110

0

$$n = n-1 = 3-1 = 2$$

Step-4

AC

Q

Q₋₁

operation

00101

10110

0

ASR

00

00101
↓ ↓ ↓ ↓
00010

10110
↓ ↓ ↓ ↓
11011

0 → Ignore
0

ASR

00010

11011

0

$$n = n-1 = 2-1 = 1$$

Step-5

AC

Q

Q₋₁

operation

00010

11011

0

AC = AC - M

10

$$\begin{array}{r} AC = 00010 \\ 10001 \\ \hline 10011 \end{array}$$

10011
↓ ↓ ↓ ↓
11001

11011
↓ ↓ ↓ ↓
11101

0 → Ignore
1

ASR

ASR

11001

11101

1

$$n = n-1 = 1-1 = 0$$

Result $\Rightarrow R = AC, Q$

11001 11101
merge

110011101

M.S

1's complement of $R = 0011000010$

2's complement of $(-R) = +$

$$\begin{array}{r} 1 \\ \hline 0011000011 \\ \hline \end{array}$$

$$(-R) = 0011000011$$

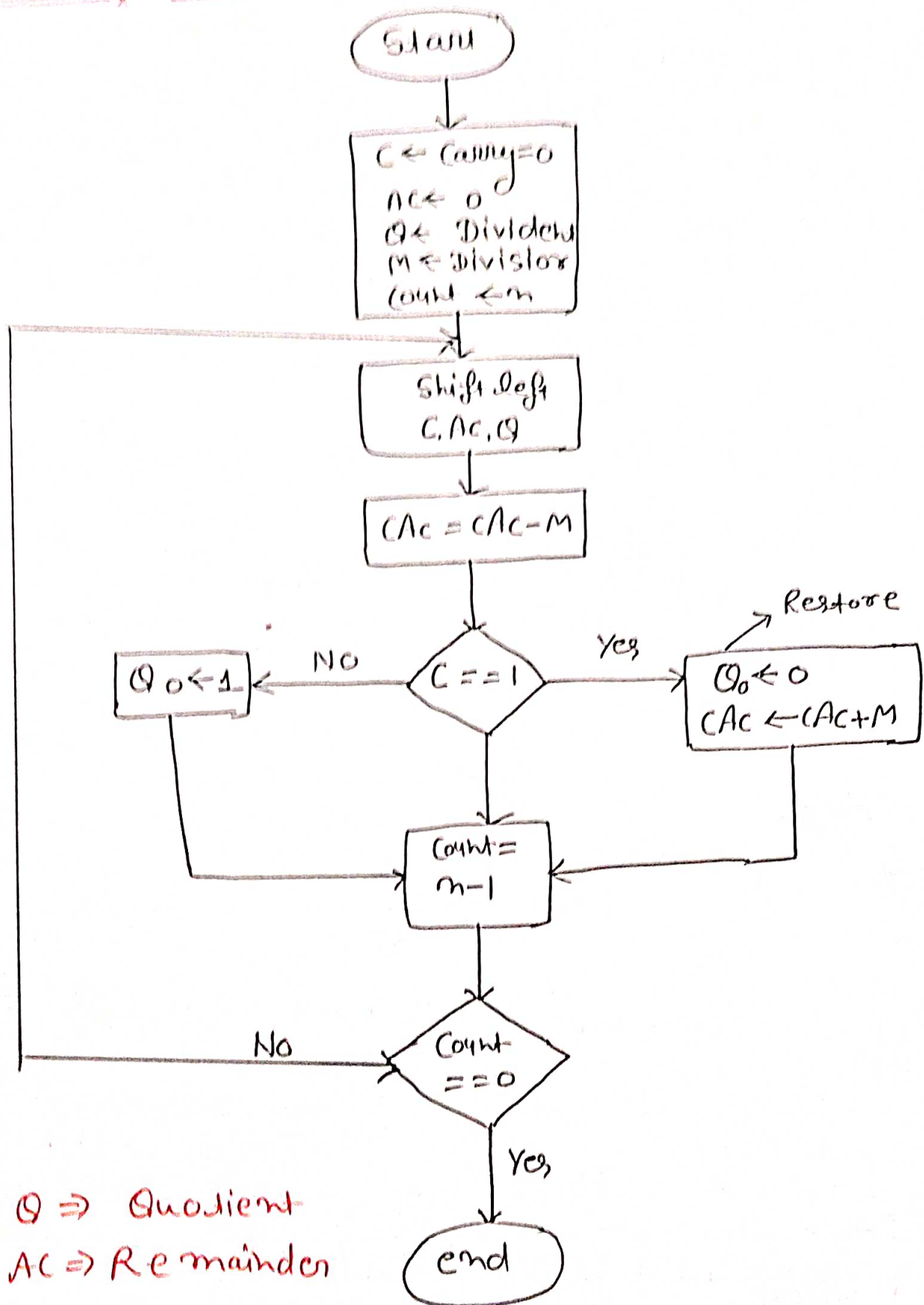
$$R = (-195)_{10} \text{ Ans}$$

Ex Show step by step the multiplication process using Booth algorithm when $(+15)$ and $(+13)$ numbers are multiplied. Assume 5-bit registers that hold signed numbers.

Ex Show step by step the multiplication process using Booth Algorithm when (-15) and (-13) numbers are multiplied. Assume 5-bit registers that hold signed numbers.

Do yourself

Restoring Division method: FLOW CHART



$Q \Rightarrow$ Quotient
 $AC \Rightarrow$ Remainder

Q1 Perform Division of 40 following $12/3$.

Sol $\theta = 12$

$M = 3$

$\theta = 12 \Rightarrow (1100)_2$

$M = 3 \Rightarrow (00011)_2$
add

1's complement of $M = 11100$

2's complement of $(-M) = \begin{array}{r} + \\ 11101 \end{array}$

$(-M) = (11101)_2$

$AC = 0000 \quad n = 4$

Step 1:

C	AC	θ	operation
0	0000	1100	Shift left
0	0001	100 	$CAC = CAC - M$ $ \begin{array}{r} = 00001 \\ 11101 \\ \hline 11110 \end{array} $
1	1110	100 0	$CAC = CAC + M$ $ \begin{array}{r} = 11110 \\ 00011 \\ \hline 100001 \end{array} $
M.S	1110	100 0	discard 1 $n = n - 1 = 4 - 1$ $= 3$

Step-2:

C	AC	S	Operation
0	0001	1000	Shift left
0	0011	000□	$CAC = CAC - M$ $= \begin{array}{r} 00011 \\ 11101 \\ \hline 100000 \end{array}$ Discard carry $n = 3 - 1$ $= 2$
0	0000	000□1	

Step-3:

C	AC	S	Operation
0	0000	0001	Shift left
0	0000	001□	$CAC = CAC - M$ $= \begin{array}{r} 00000 \\ 11101 \\ \hline 11101 \end{array}$
1	1101	001□0	$CAC = CAC + M$ $= \begin{array}{r} 11101 \\ 00011 \\ \hline 100000 \end{array}$ Discard $n = 2 - 1$ $= 1$
0	0000	001□0	

M.S

Step-4'

C	AC	Q	operation
0	0000	0010	Shift left
0	0000	010□	$AC = AC - M$
			$= \begin{array}{r} 00000 \\ 11101 \\ \hline 11101 \end{array}$
1	1101	010□0	$AC = AC + M$
			$= \begin{array}{r} 11101 \\ 00011 \\ \hline 100000 \end{array}$
0	0000	010□0	$\begin{array}{r} 100000 \\ \downarrow \text{Discard} \\ n = 1 - 1 \\ = 0 \end{array}$

AC = Remainder $\Rightarrow 0$

Q = 0100

Q = $(4)_{10}$ Ans

Q2 Perform division of the following $17/3$.

Sol

Q = 17 M = 3 n = 5

Q = $(17)_{10}$

$\Rightarrow (10001)_2$

M = $(3)_{10} \Rightarrow (11)_2$

M = $\begin{array}{c} \rightarrow 000011 \\ C \end{array}$

M.S

1's complement of $M = 111100$

2's complement of $M = + \frac{1}{111101}$

$$(-M) = (111101)_2$$

Step-1

C	AC	Q	operation $n=5$
0	00000	10001	Shift-Left
0	00001	0001 □	$CAC = CAC - M$ $= \begin{array}{r} 000001 \\ 111101 \\ \hline 111110 \end{array}$
1	11110	0001 □	$CAC = CAC + M$ $= \begin{array}{r} 111110 \\ 000011 \\ \hline 100001 \end{array}$ ① Discard carry $n = 5 - 1 = 4$
0	00001	00010	

Step-2

C	AC	Q	Operation $n=4$
0	00001	00010	Shift-Left M.S

$$\begin{array}{r}
 0 \quad 00010 \quad 0010 \boxed{} \quad C_{Ac} = C_{Ac} - M \\
 = 000010 \\
 \begin{array}{r}
 111101 \\
 \hline
 111111
 \end{array} \\
 1 \quad 11111 \quad 0010 \boxed{0} \quad C_{Ac} = C_{Ac} + M
 \end{array}$$

$$\begin{array}{r}
 = 111111 \\
 000011 \\
 \hline
 \textcircled{1} 000010 \\
 \downarrow \\
 \text{Discard carry} \\
 n = 4 - 1 = 3
 \end{array}$$

$$0 \quad 00010 \quad 00100$$

Step-3

C A_C Q operation n=3

$$0 \quad 00010 \quad 00100 \quad \text{Shift left}$$

$$0 \quad 00100 \quad 0100 \boxed{} \quad C_{Ac} = C_{Ac} - M$$

$$\begin{array}{r}
 = 000100 \\
 111101 \\
 \hline
 \textcircled{1} 000001
 \end{array}$$

$$0 \quad 00001 \quad 0100 \boxed{1}$$

$$\begin{array}{r}
 \text{Discard carry} \\
 n = 3 - 1 \\
 = 2
 \end{array}$$

Step-4

C A_C Q operation

$$0 \quad 00001 \quad 01001 \quad \text{Shift left}$$

$$0 \quad 00010 \quad 1001 \boxed{} \quad C_{Ac} = C_{Ac} - M$$

M.S

$$\begin{array}{r}
 = 000010 \\
 111101 \\
 \hline
 111111
 \end{array}$$

$$1 \quad 11111 \quad 1001 \boxed{0} \quad CAC = CAC + M$$

$$\begin{array}{r}
 = 111111 \\
 000011 \\
 \hline
 \textcircled{1} 000010
 \end{array}$$

↓ Discard carry
n = 2 - 1 = 1

$$0 \quad 00010 \quad 10010$$

Step-5

C	AC	Q	operation
0	00010	10010	Shift Left
0	00101	0010 $\square$	$CAC = CAC - M$

$$\begin{array}{r}
 = 000101 \\
 111101 \\
 \hline
 \textcircled{1} 000010
 \end{array}$$

↓ Discard carry
n = 1 - 1 = 0

$$1 \quad 00010 \quad 0010 \boxed{1}$$

$$Q = 00101 = (5)_{10}$$

$$AC = 00010 = (2)_{10}$$

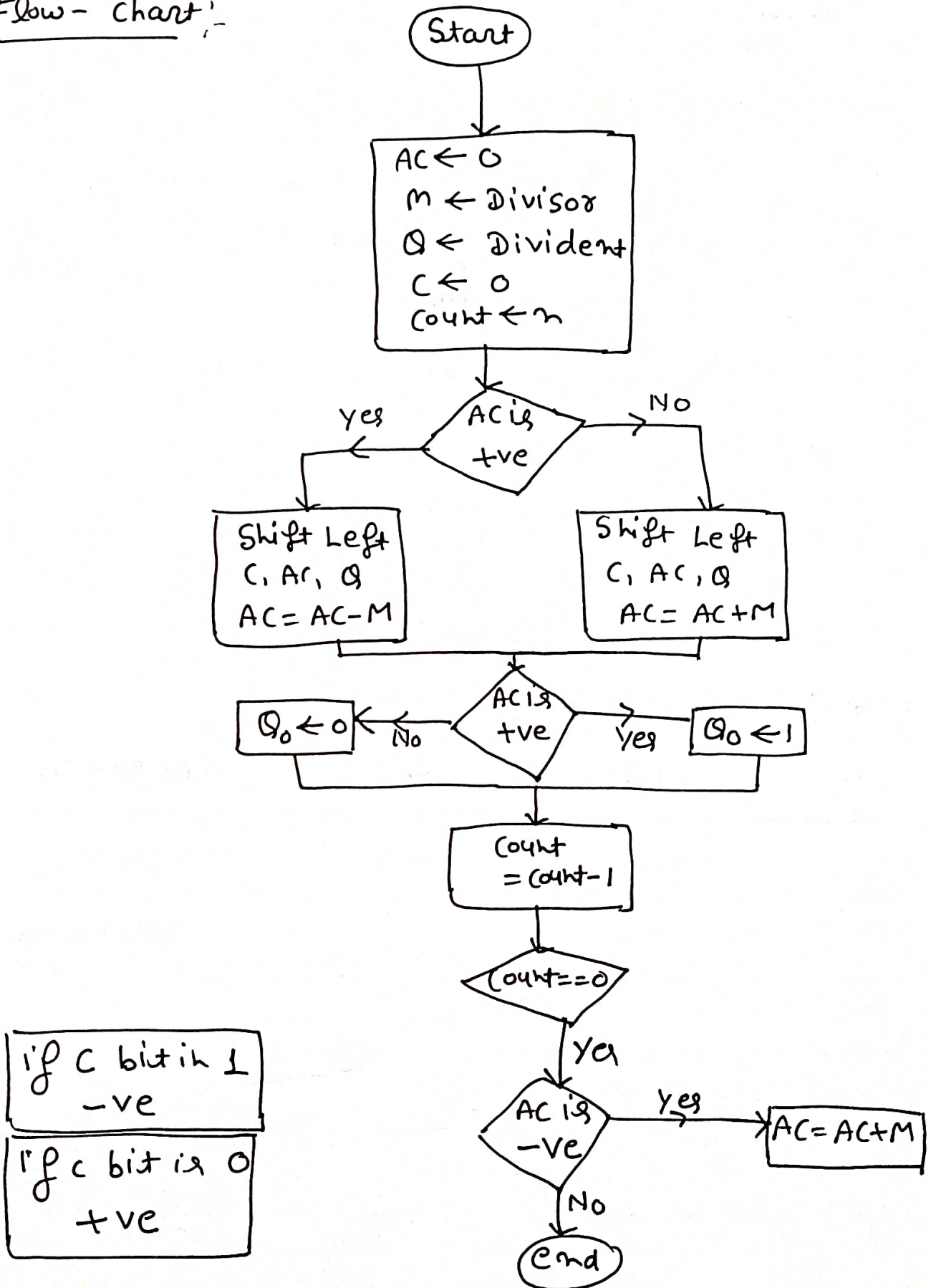
$$\text{Remainder} = (2)_{10}$$

$$\text{Quotient} = (5)_{10} \quad \underline{\underline{\text{Ans}}}$$

M.S

Non-RESToring Division method:-

Flow-chart:-



Q₁ Perform division using non-restoring division $(10)_{10} \div (3)_{10}$

Sol $\theta = (10)_{10} \Rightarrow (1010)_2$

$M = (3)_{10} \Rightarrow (11)_2 \quad n = 4$

$\begin{matrix} & 0 & 0 & 0 & 1 & 1 \\ & \nearrow & & & & \\ C(\text{bit}) & & & & & \end{matrix}$

$M = (00011)_2$

1's complement of $M = 11100$

2's complement of $(-M) = \begin{array}{r} 11100 \\ + \quad 1 \\ \hline 11101 \end{array}$

$-M = (11101)_2$

Step 1:

C	AC	Q	operation
0	0000	1010	Initially
0	0001	010 □	Shift left
0	0001	010 □	$AC = AC - M$
			$= 00001$
			$\begin{array}{r} 00001 \\ 11101 \\ \hline \textcircled{1}1110 \end{array}$
1	1110	010 0	$\begin{matrix} \nearrow \\ C(\text{bit}) \end{matrix} \quad \begin{matrix} n = h-1 \\ = 4-1 \\ = 3 \end{matrix}$

Step-2

C	AC	Q	operation
1	1110	0100	Shift Left
1	1100	100 	$AC = AC + M$ $= \begin{array}{r} 11100 \\ 00011 \\ \hline \end{array}$ $\textcircled{1} 1111$ (C bit) $m = m-1$ $= 3-1 = 2$
1	1111	100 0	

Step 3:

C	AC	Q	operation
1	1111	1000	Shift Left
1	1111	000 	$AC = AC + M$ $= \begin{array}{r} 11111 \\ 00011 \\ \hline \end{array}$ $\textcircled{1} 00010$ $\downarrow$ Discard carry $m = m-1 = 2-1 = 1$
0	0010	000 11	

Step 4:

C	AC	Q	operation
0	0010	0001	Shift Left
0	0100	001 	$AC = AC - M$ $= \begin{array}{r} 00100 \\ 11101 \\ \hline \end{array}$ $\textcircled{1} 00001$ Discard carry $m = m-1 = 1-1 = 0$
0	0001	001 1	

Remainder = C, AC Quotient = Q

$$R = (00001)_2$$

$$Q = (0011)_2$$

$$= (1)_{10}$$

$$Q = (3)_{10}$$

$$\text{Quotient} = (3)_{10} \quad 10 \div 3$$

$$\text{Remainder} = (1)_{10} \quad \text{Ans}$$

Q2 Divide $(1011)_2$ with $(0011)_2$ using non-restoring division methods.

sol

$$Q = (1011)_2$$

$$M = (0011)_2$$

$$M = (0001)_2$$

cc bit

is complement of $M = 11100$

$$2's \text{ complement of } (-M) = \begin{array}{r} 1 \\ \hline 11101 \end{array}$$

$$-M = (11101)_2$$

$$Q = (1011)_2$$

$$n \text{ no of bit} = 4$$

Step-1

C	AC	Q	Operations
0	0000	1011	Initial
0	0001	011 	Shift left
			$AC = AC - M$
			$= \begin{array}{r} 00001 \\ 11101 \\ \hline 11110 \end{array}$
1	1110	011 0	$n = 4 - 1 = 3$

Step-2

C	AC	Q	Operations
1	1110	0110	Shift left
1	1100	110 	$AC = AC + M$
			$= \begin{array}{r} 11100 \\ 00011 \\ \hline 11111 \end{array}$
1	1111	110 0	$n = n - 1$ $= 3 - 1$ $= 2$

Step-3

C	AC	Q	Operations
1	1111	1100	Shift left
1	1111	100 	$AC = AC + M$
			$= \begin{array}{r} 11111 \\ 00011 \\ \hline 10010 \end{array}$
0	0010	100 1	① 00010 ↓ Discard carry
			$n = n - 1 = 2 - 1$ $= 1$

Step	C	AC	Q	Operation
0		0010	1001	Shift Left
0		0101	001 □	$AC = AC - M$ $= 00101$ 11101 00010 Discard carry $n = m - 1$ $= 1 - 1 = 0$
0		0010	001 □	

Remainder = C, AC

$$R = (00010)_2$$

$$= (2)_{10}$$

Quotient = Q

$$Q = (0011)_2$$

$$Q = (3)_{10}$$

$$Q = (3)_{10}$$

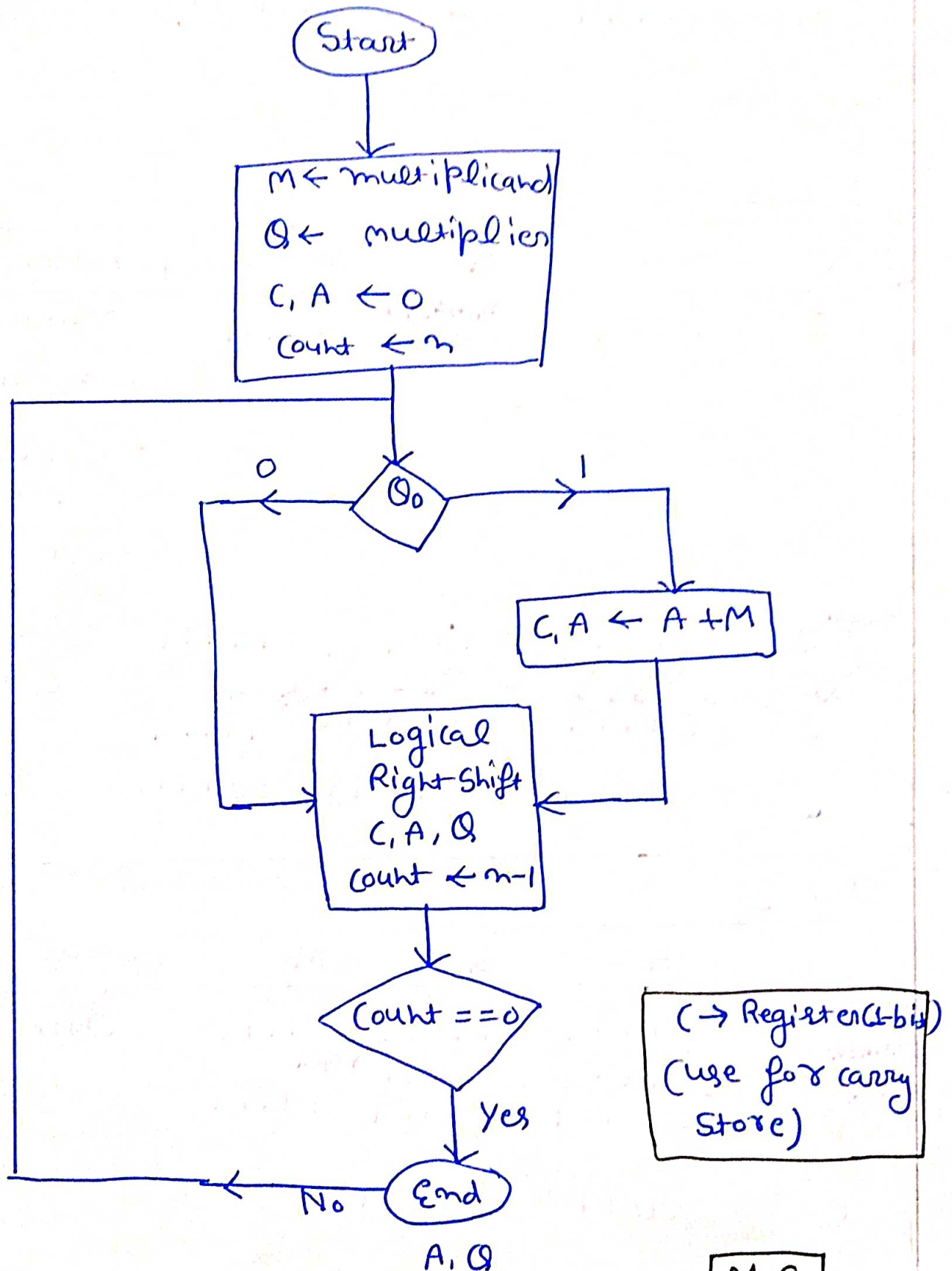
$$(11)_{10} \div (3)_{10}$$

$$Q = (3)_{10}$$

$$R = (2)_{10} \quad \underline{\underline{\text{Ans}}}$$

Unsigned number multiplication:-

FLOW CHART FOR UNSIGNED MULTIPLICATION



Ex-1 Show the contents of the Registers E, A, Q, SC during Process of multiplication of two binary no.

$$M = 11111 \quad Q = 10101 \quad n = 5$$

Sol

Step-1

C	A	Q	M	operations
0	00000	^{Q₄ Q₃ Q₂ Q₁ Q₀} 10101	11111	Initially C, A ← A + M = 00000 11111 ----- 01111
0	01111	11010	11111	Shift Right C, A, Q $n = n - 1$ $= 5 - 1$ $= 4$

Diagram for Step 1: An 'Insert' circle with an arrow points to the first '0' in the C register. Arrows point from the last four bits of the A register (1111) to the first four bits of the Q register (1101). An arrow points from the last bit of the Q register (0) to the last bit of the M register (1). A label 'Last' is next to the arrow from Q to M.

Step-2

C	A	Q	M	operations
0	00111	11101	11111	Shift right $n = 4 - 1$ $= 3$

Diagram for Step 2: An 'Insert' circle with an arrow points to the first '0' in the C register. Arrows point from the last three bits of the A register (011) to the first three bits of the Q register (111). An arrow points from the last bit of the Q register (1) to the last bit of the M register (1). A label 'Ignore' is next to the arrow from Q to M.

Step-3

C	A	Q	M	operation
0	00111	11101	11111	$C, A = A + M$
				$ \begin{array}{r} 000111 \\ 11111 \\ \hline 100110 \end{array} $
1 Insert 0	00110 ↓ ↓ ↓ ↓ 10011	11101 ↓ ↓ ↓ ↓ 01110	11111 ↓ ↓ ↓ ↓ 11111	Shift-Right C, A, Q
0	10011	01110	11111	$n = n - 1$ $= 3 - 1$ $= 2$

Step-4

C	A	Q	M	operations
0 Insert 0	10011 ↓ ↓ ↓ ↓ 01001	01110 ↓ ↓ ↓ ↓ 10111	11111 ↓ ↓ ↓ ↓ 11111	Shift Right $n = n - 1$ $= 2 - 1$ $= 1$

M.S

Step-5

C	A	Q	M	operation
0	01001	10111	11111	$C, A = A + M$
				$ \begin{array}{r} 001001 \\ 1111 \\ \hline 101000 \end{array} $
Insert ↓ 0	01000	10111	11111	Shift Right
	↓ ↓ ↓ ↓	↓ ↓ ↓ ↓	↓ ↓ ↓ ↓	$n = 1 - 1 = 0$
	10100	01011	11111	

Result $R = A, Q$

$= 10100 \ 01011$

merge

$= 1010001011$

$R \Rightarrow (65D)_{10}$ Ans

Ex-2 Show the contents of the Registers
E, A, Q, SC during Process of
multiplication of two binary No
 $M = 1011$ $Q = 1101$ $n = 4$