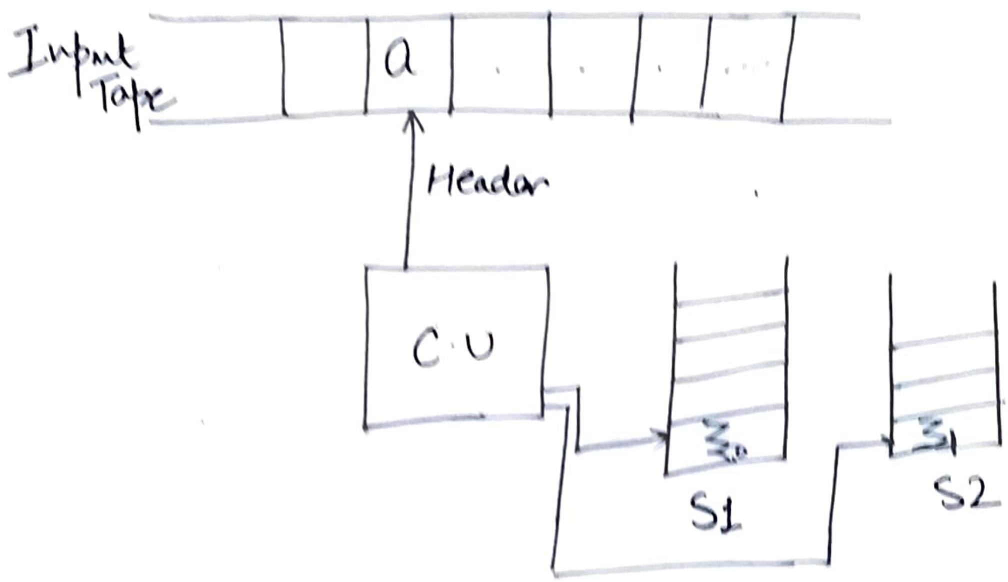
Two - Stack PDA :

FA $\longleftrightarrow$ RL $\{a^n / n \geq 0\}$

PDA $\longleftrightarrow$ CFL $\{a^n b^n / n \geq 0\}$

2S-PDA $\longleftrightarrow$ CSL $\{a^n b^n c^n / n \geq 0\}$



$$M = (Q, \Sigma, \Gamma_1, \Gamma_2, \delta, q_0, z_0, Z_1, F)$$

Q = is the set of finite states

Σ = is the input alphabet

Γ_1 = is the Stack 1 alphabet

Γ_2 = " " stack 2 "

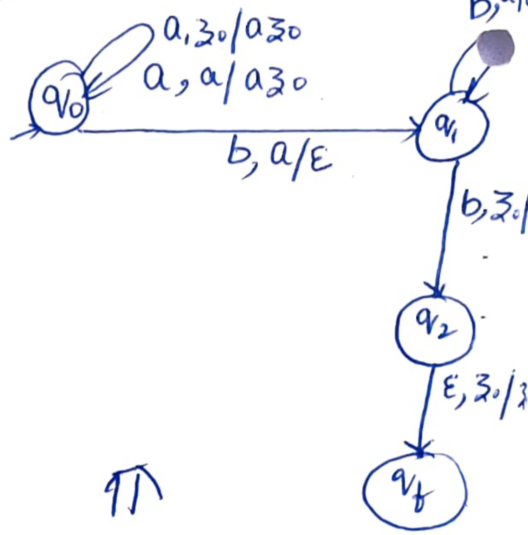
q_0 = " " initial state $q_0 \in Q$

Z_0 = initial symbol of Stack 1, $Z_0 \in \Gamma_1$

Z_1 = ,, ,, Stack 2, $Z_1 \in \Gamma_2$

F = is the final state

$$\boxed{\delta = Q \times \Sigma \times \Gamma_1 \times \Gamma_2 \longrightarrow Q \times \Gamma_1^* \times \Gamma_2^*}$$



⇑

$a^n b^{n+1}$

* $a^n b^n$
* $a^n b^{2n}$
* wcw^R
* $|w|_a = b$ / $w \in \{a, b\}$
* $a^{m+n} b^m c^n$
* $a^n b^{m+n} c^n$
* $a^n b^m c^n$

$a^n b^n c^m$

$a^n b^m c^{m+n}$

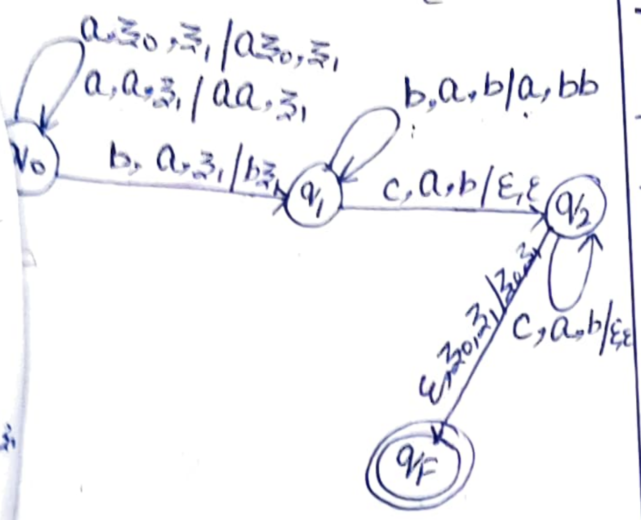
→ * $a^n b^m c^m$

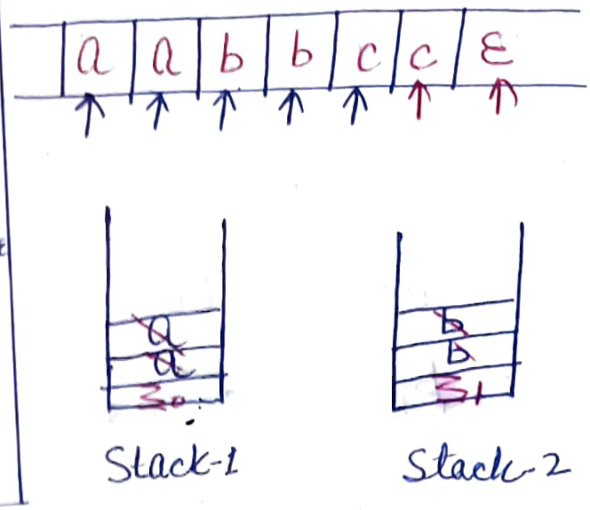
→ * $a^n b^{n+1}$

Design a Two-Stack PDA for the Language
$L = \{a^n b^n c^n / n \geq 0\}$

Solution:

$L = \{abc,\ aabbcc,\ aaabbbccc \ldots\}$





$\delta(q_0, a, z_0, z_1) = (q_0, az_0, z_1)$

$\delta(q_0, a, a, z_1) = (q_0, aa, z_1)$

$\delta(q_0, b, a, z_1) = (q_1, a, bz_1)$

$\delta(q_1, b, a, b) = (q_1, a, bb)$

$\delta(q_1, c, a, b) = (q_2, \varepsilon, \varepsilon)$

$\delta(q_2, c, a, b) = (q_2, \varepsilon, \varepsilon)$

$\delta(q_2, \varepsilon, z_0, z_1) = (q_F, z_0, z_1)$

if $n = 0$

$\delta(q_0, \varepsilon, z_0, z_1) = (q_f, z_0, z_1)$

$M = (Q, \Sigma, \Gamma_1, \Gamma_2, \delta, q_0, F, z_0, z_1)$

$Q = \{q_0, q_1, q_2, q_F)$

$\Sigma = \{a, b, c\}$

$\Gamma_1 = \{a, z_0\}$

$\Gamma_2 = \{b, z_1\}$

$q_0 = q_0$

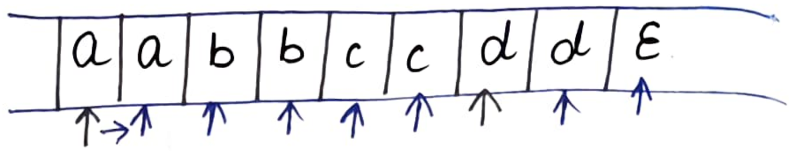
$z_0 = z_0$

$z_1 = z_1$

$F = q_F$

Design a Two-Stack PDA for $\{a^n b^n c^n d^n / n \in N\}$

$L = \{abcd, aabbccdd, aaabbbcccddd\}$



$\delta(q_0, a, z_0, z_1) = (q_0, az_0, z_1)$

$\delta(q_0, a, a, z_1) = (q_0, aa, z_1)$

$\delta(q_0, b, a, z_1) = (q_1, bz_1, \varepsilon)$

$\delta(q_1, b, a, b) = (q_1, bb, \varepsilon)$

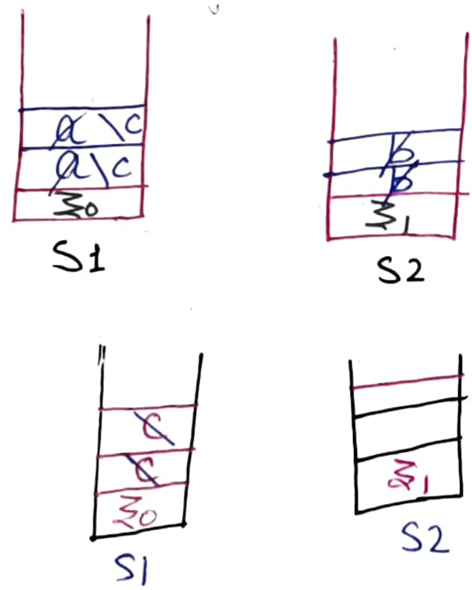
$\delta(q_1, c, b, z_0) = (q_2, cz_0, \varepsilon)$

$\delta(q_2, c, c, b) = (q_2, cc, \varepsilon)$

$\delta(q_2, d, c, z_1) = (q_3, z_1, \varepsilon)$

$\delta(q_3, d, c, z_1) = (q_3, \varepsilon, z_1)$

$\delta(q_3, \varepsilon, z_0, z_1) = (q_F, z_0, z_1)$



$M = \{Q, \Sigma, \Gamma_1, \Gamma_2, \delta, q_0, F, z_0, z_1\}$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b, c, d\}$

$\Gamma_1 = \{z_0, a, c\}$

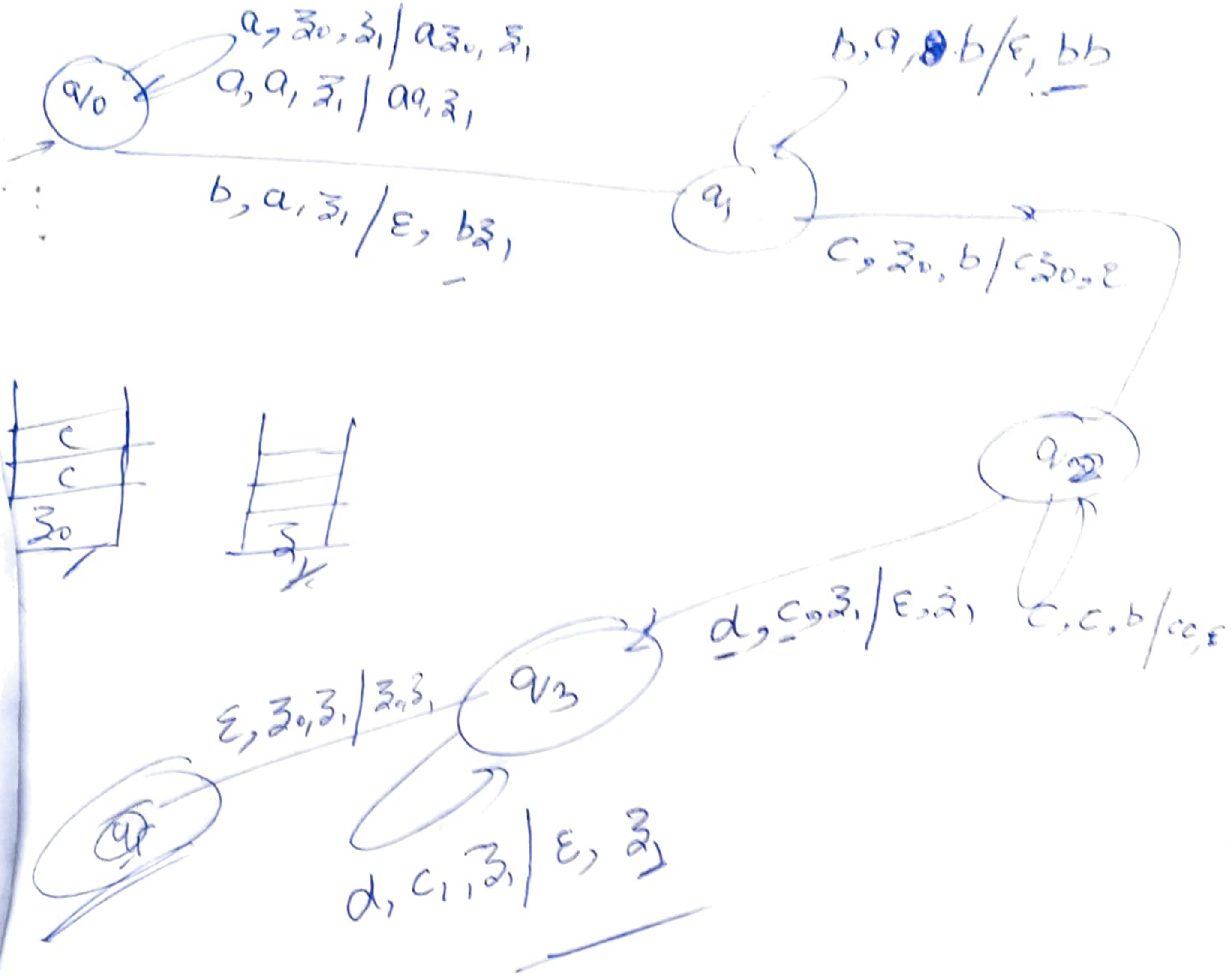
$\Gamma_2 = \{z_1, b\}$

$z_0 = \{z_0\}$

$z_1 = \{z_1\}$

$F = q_F$

$q = q_0$



q_0
$a, z_0, z_1 / a z_0, z_1$
$a, a, z_1 / aa, z_1$
$b, a, z_1 / \varepsilon, b z_1$
q_1
$b, a, b / \varepsilon, bb$
$c, z_0, b / c z_0, \varepsilon$
q_2
$c, c, b / cc, \varepsilon$
$d, c, z_1 / \varepsilon, z_1$
q_3
$d, c, z_1 / \varepsilon, z_1$
$\varepsilon, z_0, z_1 / z_0 z_1$
q_f
c
c
z_0
z_1

PDA to CFG :

To achieve PDA to CFG, following rules will be used,

Triplets $\rightarrow S \cup [q, A, P]$, $q, P \in Q$, $A \in \Gamma$

1. $\boxed{S \rightarrow [q_0, z_0, P] \text{ for each } P}$

2. if $\delta(q, x, A) = (P, B_1 B_2 \cdots B_m)$ → stack symbols

$[q, A, q_{m+1}] \rightarrow x [P, B_1, q_2][q_2, B_2, q_3] \cdots\cdots [q_m, B_m, q_{m+1}]$

3. if $\delta(q, x, A) = (P, \varepsilon)$

$[q, A, P] \rightarrow x$

$\boxed{x \in (\Sigma \cup \{\varepsilon\})}$

Example:

Convert the following PDA into its equivalent CFG

1. $\delta(q_0, a, z_0) = (q_0, Xz_0)$
2. $\delta(q_0, a, X) = (q_0, XX)$
3. $\delta(q_0, b, X) = (q_1, \varepsilon)$
4. $\delta(q_1, b, X) = (q_1, \varepsilon)$
5. $\delta(q_1, \varepsilon, z_0) = (q_1, \varepsilon)$

Solution:

From the above PDA we ma[y] write the following values

$Q = \{q_0, q_1\}$	δ	q_0
$\Sigma = \{a, b\}$	$\Gamma = \{z_0, X\}$	z_0
		$\phi = F$

It may be represented as follows

$M = (\{q_0, q_1\}, \{a, b\}, \delta, \{z_0, X\}, q_0, z_0, \phi)$

Since, $G = \{\Sigma, V_n, P, S\}$

$\Sigma \rightarrow$ F.S.O. Terminal

$V_n \rightarrow$ Non-e S. of Non-terminals

$P \rightarrow$ Production Rules

$S \rightarrow$ start symbol

From the rule 1, following is the productions

$S \rightarrow [q_0, z_0, \boxed{q_0}]$, $S \rightarrow [q_0, z_0, \boxed{q_1}]$

Using the rule number-2, we may write the following productions for transition-1

$x[q_0, z_0, q_0] \longrightarrow a[q_0, X, q_0][q_0, z_0, q_0]$

$x[q_0, z_0, q_0] \longrightarrow a[q_0, X, q_1][q_1, z_0, q_0]$

$[q_0, z_0, q_1] \longrightarrow a[q_0, X, q_0][q_0, z_0, q_1]$

$[q_0, z_0, q_1] \longrightarrow a[q_0, X, q_1][q_1, z_0, q_1]$

using the rule number-2, following productions may obtained for transition 2

$x[q_0, X, q_0] \longrightarrow a[q_0, X, q_0][q_0, X, q_0]$

$x[q_0, X, q_0] \longrightarrow a[q_0, X, q_1][q_1, X, q_0]$

$[q_0, X, q_1] \longrightarrow a[q_0, X, q_0][q_0, X, q_1]$

$[q_0, X, q_1] \longrightarrow a[q_0, X, q_1][q_1, X, q_1]$

using rule number-3, following production is obtained for transition-3

$[q_0, X, q_1] \rightarrow b$

using rule 3, following production is obtained for transition-4

$[q_1, X, q_1] \rightarrow b$

using rule 3, following production is obtained for transition-5

$[q_1, z_0, q_1] \Rightarrow \varepsilon$

All the above obtained productions not the productive productions, we need to eliminate all the non-prod productions using the following method.

Remove/eliminate all those produc which are not present in LHS bu present in RHS.

$\{q_1, z_0, q_0\}$; $\{q_1, X, q_0\}$

Now, lets rename all the obtained transition in the following way,

$q_0, z_0, q_0 = A$

$q_0, X, q_0 = B$

$q_0, X, q_1 = C$

$q_1, z_0, q_0 = D$

$q_0, z_0, q_1 = E$

$q_1, X, q_0 = F$

$q_1, X, q_1 = G$

$q_1, z_0, q_1 = H$

After renaming obtained productions will be as follows,

$S \to A$
$S \to E$
$A \to aBA$
$A \to aCD$
$E \to aBE$
$E \to aCE$
$B \to aBB$
$B \to aCF$
$C \to aBC$
$C \to aCG$
$C \to b$
$G \to b$
$H \to \varepsilon$

$S \to A / E$
$A \to aBA / aCD$
$E \to aBE / aCE$
$B \to aBB / aCF$
$C \to aBC / aCG / b$
$G \to b$
$H \to \varepsilon$

After eliminating useless productions, we get the following CFG

$S \rightarrow E$

$E \rightarrow aCE$

$C \rightarrow aCG$

$C \rightarrow b$

$G \rightarrow b$

$H \rightarrow \varepsilon$

Examples:

Construct equivalent CFG for the following PDA

1 $\delta(q_0, 1, z_0) = (q_0, Xz_0)$
2 $\delta(q_0, 1, X) = (q_0, XX)$
3 $\delta(q_0, 0, X) = (q_1, X)$
4 $\delta(q_1, 1, X) = (q_1, \varepsilon)$
5 $\delta(q_1, 0, z_0) = (q_0, z_0)$
6 $\delta(q_0, \varepsilon, z_0) = (q_0, \varepsilon)$

Solution:

PDA is defined by following tuples,

$M = (Q, \Sigma, \delta, q_0, F, \Gamma, z_0)$

where, $Q = \{q_0, q_1\}$, $\Sigma = \{0, 1\}$,

$\Gamma = (z_0, X)$, q_0, z_0, ϕ

CFG is defined by following 4 tuples,

$M = (\Sigma, V_n, P, S\}$

To obtain equivalent CFG following rules will be used

$S \cup [q, A, p]$, $q, p \in Q$, $A \in \Gamma$

1. $S \rightarrow [q_0, z_0, p]$, for each p

2. if $\delta(q, x, A) = (p, B_1 B_2 \ldots B_m)$ then,

$$[q, A, q_{m+1}] \rightarrow x [p, B_1, q_2] [q_2, B_2, q_3] \ldots [q_m, B_m, q_{m+1}]$$

3. if $\delta(q, x, A) = (p, \varepsilon)$ then,

$$[q, A, p] \rightarrow x$$

From the rule 1, following are the start productions,

$$S \rightarrow [q_0, z_0, q_0],\ S \rightarrow [q_0, z_0, q_1]$$

P.T.O

Using the rule 2, following productions are obtained for transition-1

$[q_0, Z_0, q_0] \rightarrow 1 [q_0, X, q_0] [q_0, Z_0, q_0]$

$[q_0, Z_0, q_0] \rightarrow 1 [q_0, X, q_1] [q_1, Z_0, q_0]$

$[q_0, Z_0, q_1] \rightarrow 1 [q_0, X, q_0] [q_0, Z_0, q_1]$

$[q_0, Z_0, q_1] \rightarrow 1 [q_0, X, q_1] [q_1, Z_0, q_1]$

Using the rule 2, following productions are obtained for transition-2

$[q_0, X, q_0] \rightarrow 1 [q_0, X, q_0] [q_0, X, q_0]$

$[q_0, X, q_0] \rightarrow 1 [q_0, X, q_1] [q_1, X, q_0]$

$[q_0, X, q_1] \rightarrow 1 [q_0, X, q_0] [q_0, X, q_1]$

$[q_0, X, q_1] \rightarrow 1 [q_0, X, q_1] [q_1, X, q_1]$

Using the rule 2, following productions are obtained for transition-3

$[q_0, X, q_0] \rightarrow 0 [q_1, X, q_0]$

$[q_0, X, q_1] \rightarrow 0 [q_1, X, q_1]$

using rule 3, following productions are obtained for transition - 4

$[q_1, X, q_1] \longrightarrow 1$

using rule 2, following productions are obtained for transition - 5

$[q_1, z_0, q_0] \longrightarrow 0\,[q_0, z_0, q_0]$

$[q_1, z_0, q_1] \longrightarrow 0\,[q_0, z_0, q_1]$

using rule 3, following productions are obtained for transition - 6

$[q_0, z_0, q_0] \longrightarrow \varepsilon$

Now, rename all the obtained productions as follows

$[q_0, z_0, q_0] = A$

$[q_0, z_0, q_1] = B$

$[q_1, z_0, q_0] = C$

$[q_1, z_0, q_1] = D$

$[q_0, X, q_0] = E$

$[q_0, X, q_1] = F$

$[q_1, X, q_0] = G$

$[q_1, X, q_1] = H$

After substituting, we get the following productions

$S \to A$

$S \to B$

$A \to 1EA$

$A \to 1FC$

$A \to \varepsilon$

$B \to 1EB$

$B \to 1FD$

$E \to 1EF$

$E \to 1FG$

$E \to 0G$

$F \to 1EF / 1FH / 0H$

$H \to 1$

$C \to 0A$

$D \to 0B$

After eliminating useless productions, we get the following

$S \to A$

$A \to 1FC / \varepsilon$

$F \to 1FH / 0H$

$H \to 1$

$C \to 0A$

Pumping lema for Context free language

Pumping lema is used to prove that a language is not context free.

Let 'L' be a CFL. Let 'n' be a integer constant. Select a string 'z' form L such that $\boxed{|z| \geq n}$. → length of the string is greater than or equal to.

Divide the string z into 5 parts 'uvwxy' such that

1. $|vwx| \leq n$ → Less than
2. $|vx| \geq 1$ → greater than

for $i \geq 0$, uv^iwx^iy is in the language

Example: Show that $L = \{a^n b^n c^n \mid n \geq 1\}$ is not a CFL.

Solution:

Let 'L' be a C.F.L $L = \{a^n b^n c^n, n \geq 1\}$

$L = \{abc, aabbcc, aaabbbccc \ldots\}$

Lets take $n = 3$

$z = aaabbbccc$

$|z| \geq n$

$9 \geq n \rightarrow$ True

$\underbrace{aa}_{u}\underbrace{a}_{v}\underbrace{b}_{w}\underbrace{b}_{x}\underbrace{bccc}_{y}$

Example: Prove that $L = \{a^p \mid p \text{ is a prime number}\}$ is not CFL

Solution: Let 'L' be a CFL,

$L = \{aa, aaa, aaaaa, aaaaaaa, ..\}$

Lets take $n = 5$, so

$z = aaaaa$, $|z| \geq n$

$\boxed{5 \geq 5} \rightarrow$ condition satisfied

Now divide 'z' into five parts

$\underbrace{a}_{u}\ \underbrace{a}_{v}\ \underbrace{a}_{w}\ \underbrace{a}_{x}\ \underbrace{a}_{y}$

Now check the conditions

1. $|vwx| \leq n$

$|a \cdot a \cdot a| \leq 5$

$3 \leq 5 \rightarrow$ condition satisfied

2. $|vx| \geq 1$

$|aa| \geq 1$

$2 \geq 1 \rightarrow$ condition satisfied

Now pump the string

$i = 0$, $a a^0 a a^0 a \Rightarrow \boxed{a \cdot a \cdot a \in L}$

$i = 1$ $a a^1 a a^1 a \Rightarrow aaaaa \in L$

$i = 2$ $a \cdot a^2\ a a^2 a \Rightarrow aaaaaaa \in L$

$i = 3$ $aa^3 \cdot aa^3 a$

$aaaaaaaaa \notin L$

Hence given La is not CFL

Check the given conditions

1. $|vwx| \le n$

$|a \cdot b \cdot b| \le n$

$3 \le 3 \rightarrow$ condition is true

2. $|vx| \geq 1$

$|a \cdot b| \geq 1$

$2 \geq 1 \rightarrow$ condition is true

for $i = 0$, $aa\,a^0\, b\, b^0\, bccc$

$\Rightarrow aa \cdot \varepsilon \cdot b \cdot \varepsilon\, bccc$

$\Rightarrow aabbccc$

The obtained language is not part of the language. Hence the given language is not CFL