UNIT 2 : Electromagnetic Theory

→ Introduction :

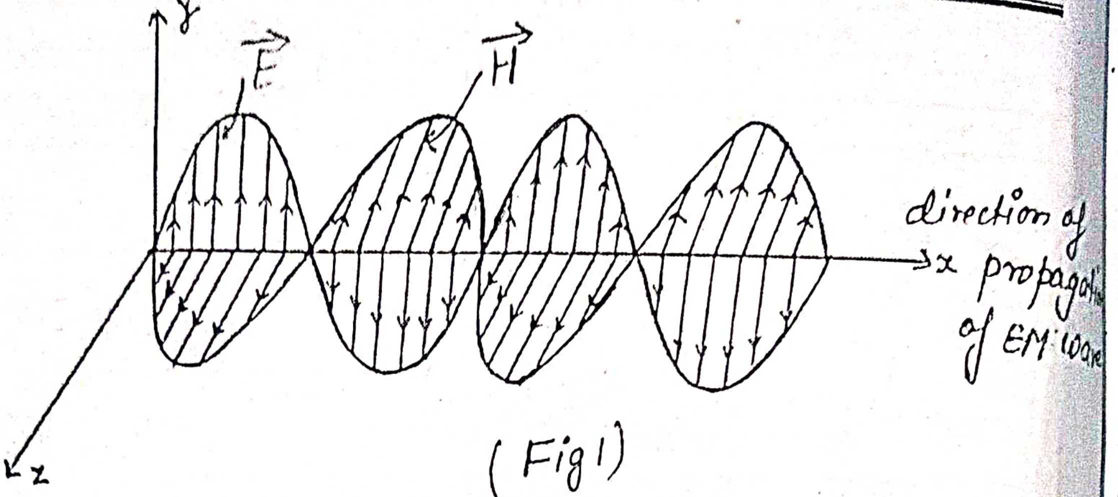
Initially, electricity and magnetism were studied separately.
(Charge at rest) (Charge in uniform motion)

In 1820, Oersted showed that magnetic field can be produced by electric current, later on Faraday invented the phenomenon of electromagnetic Induction and showed that electric current can be produced by time varying magnetic field.

In 1864, Maxwell unified both electric and magnetic field, showed that an accelerated charge particle generates Electromagnetic waves.

The coupled Oscillating electric and magnetic field that moves with the speed of light and exhibit wave behaviour is called Electromagnetic wave. Few Examples of EM wave are microwaves, infrared rays, Ultra-violet rays, X rays and γ-rays.

Note : The electric & magnetic field of Electromagnetic wave are perpendicular to each other and also perpendicular to the direction of EM waves propagation.



(Fig 1)

⇒ Some Important Points :

(1) Dell operator : $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$ (vector quantity)

(2) Laplacian Operator $\nabla^2 = \vec{\nabla}\cdot\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ (Scalar quantity)

3) Gradient of ϕ, grad $\phi = \vec{\nabla}\phi$

4) Divergence of $\vec{A}$, div $\vec{A} = \vec{\nabla}\cdot\vec{A}$

5) Curl of $\vec{A}$, curl $\vec{A} = \vec{\nabla}\times\vec{A}$

Q: State Gauss Divergence and Stoke's Theorem. Imp

Ans: Gauss Divergence theorem: Surface integral of a vector over a closed surface area is equal to volume integral of divergence of same vector over the volume enclosed by that surface area, i.e., $\boxed{\oint \vec{A}\cdot d\vec{s} = \int(\vec{\nabla}\cdot\vec{A})\,dV}$

Stoke's Theorem: Line integral of a vector over a closed loop is equal to surface integral of curl of the same vector over the surface area enclosed by that loop, i.e.,

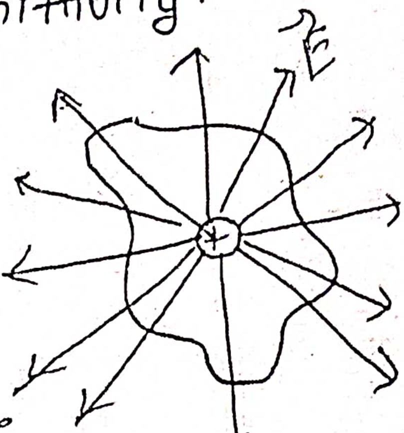
$$\boxed{\oint \vec{A}\cdot d\vec{l} = \int(\vec{\nabla}\times\vec{A})\,d\vec{S}}$$

* Basic laws of Electricity and Magnetism

1.) **Gauss law of Electrostatics:-** Gauss law of Electrostatics states that the total Electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.

$$\phi = \frac{q_{total}}{\epsilon_0}$$

$$\boxed{\oint \vec{E}\cdot dS = \frac{q_{total}}{\epsilon_0}}$$



where $\epsilon_0 \rightarrow$ permittivity of free space ($8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$) for given medium ($\epsilon$)

$\oint \vec{E}\cdot ds \rightarrow$ Electric flux

$q_{total} \rightarrow$ total charge enclosed by the body.

Here, Electric flux in an area is defined as the electric field multiplied by the area of the surface project in plane and perpendicular to the field.

2.) **Gauss law of Magnetism:-** Gauss law of Magnetism states that magnetic flux linked through any closed surface area is zero.

$$\boxed{\oint \vec{B}\cdot ds = 0}$$

Here $\oint \vec{B}\cdot ds \rightarrow$ Net magnetic flux passing through a closed surface area.

Physical significance:- (i) Magnetic monopoles do not exist.
(ii) Magnetic field lines always form a closed curve.

3) Faraday law of Electromagnetic Induction(EMI):-

(i) Whenever the magnetic flux linked with a circuit is changed, an emf is induced in the circuit.

(ii) The magnitude of induced emf is directly proportional to the negative rate of variation of magnetic flux linked with the circuit.

$$e = -\frac{d\phi_B}{dt} \quad \text{or} \quad \oint \vec{E}\cdot dl = \frac{d\phi_B}{dt}$$

where $e \rightarrow$ induced emf

$\phi_B \rightarrow$ magnetic flux linked with circuit

$\vec{E} \rightarrow$ Electric field generated in circuit due to induced emf.

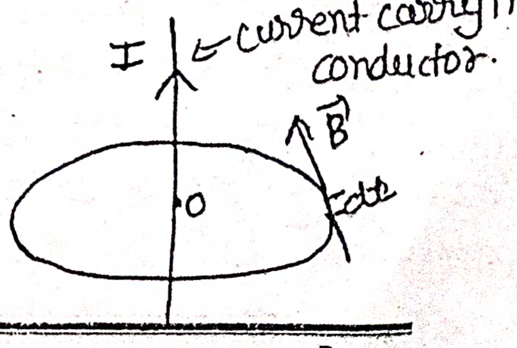
4.) Ampere's Circuit law :- According to Ampere's law the line integral of magnetic field B along a closed curve is equal to μ_0 times the net current through the area bounded by the curve.

$$\boxed{\oint \vec{B}\cdot dl = \mu_0 I_{enclosed}}$$

or

$$\boxed{\oint \vec{H}\cdot dl = I_{enclosed}}$$

Here; $\mu_0 \rightarrow$ permeability of the free space.



⇒ Continuity Equation :-

Ques : What is the equation of continuity ? Obtain the required expression for it. Also, give its physical significance. [2016-17, 2018-19]

Ans : Electric current through surface area is given by

$$I = \oint \vec{J} \cdot d\vec{s} \quad \text{---(1)} \quad (\vec{J} = \text{current density})$$

Consider, a closed surface 'S' enclosing a Volume 'V'. If 'ρ' is volume density then,

$$I = -\frac{dq}{dt} = -\frac{d}{dt}\int \rho \, dV \quad \text{------(2)}$$

If there is neither source nor sink inside volume, then according to law of conservation of charge, Eq (1) and Eq (2) will be same.

$$\therefore \quad \oint \vec{J} \cdot d\vec{s} = -\frac{d}{dt}\int \rho \, dV \quad \text{------(3)}$$

Applying Gauss Divergence theorem,

$$\int (\vec{\nabla} \cdot \vec{J}) \, dV = -\frac{d}{dt}\int \rho \, dV = -\int \frac{\partial}{\partial t} \rho \, dV$$

$$\int \left(\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} \right) dV = 0$$

For an arbitary volume, integrand must be zero, thus we have

$$\boxed{\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0} \longrightarrow \text{Equation of Continuity}$$

Physical Significance → Law of conservation of charge.

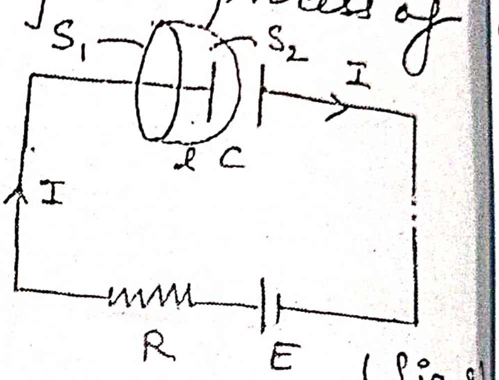
It states that total current flowing out of some volume must be equal to rate of decrease of charge within volume, assuming that charge can neither be created nor be destroyed. For steady state, $\partial \rho / \partial t = 0$

$$\therefore \quad \boxed{\vec{\nabla} \cdot \vec{J} = 0}$$

⇒ **Displacement Current:**

Ques:- Why Maxwell's proposed that Ampere law require modification? [2018-19]

Soln: The idea of modification in Ampere's law arises in connection with capacitors with no medium between them. Let us consider the circuit showing the process of charging of capacitor.



(fig 2)

Now consider two surface S_1 (plane) and S_2 (hemispherical) bounded by closed path l. Let an instant of time 't', the current is I.

Applying Ampere law for surface S_1, so we get

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \quad \text{----- (1)}$$

And, Ampere law for surface S_2 is given by,

$$\oint \vec{B} \cdot d\vec{l} = 0 \quad \text{--- (2)}$$ [∵ dielectric current inside the capacitor is zero]

We see that eqn (1) and (2) contradict each other which is impossible. So, to remove this controversy, Maxwell is introduced the modification in Ampere's law.

Ques: What is displacement current? [2015-16, 2016-17, 2018, 2020-] V. Imp

Or

Explain the concept of displacement current and show how it leads to the modification of Ampere law. [2016-17, 2018-19, 2019-20].

Ans: The concept of displacement current was introduced to resolve the paradox of charging capacitor. Maxwell proposed that

current $\xrightarrow{\text{produces}}$ Magnetic field

But, also Changing Electric field $\xrightarrow{\text{produces}}$ Magnetic field

It means that changing Electric field is equivalent to a current which flows as long as the electric field is changing. This equivalent current in vacuum or dielectric produces same magnetic field as the conduction current in conductor and is known as Displacement current.

Therefore, Maxwell introduced a factor $\varepsilon_0 \frac{d\phi_E}{dt}$ to add in eq^n (i) instead to 'I'. This term $\varepsilon_0 \frac{d\phi_E}{dt}$ is known as displacement current $[I_d]$.

Modification of Ampere's law :-

Ampere's law in differential form is given by

$$\vec{\nabla} \times \vec{H} = \vec{J} \quad \text{------ (1)}$$

The eq^n (1) stands only for steady state current but for time varying field, the current density should be modified. Thus, taking divergence of both sides of above eq^n, we get

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J}$$

$$\vec{\nabla} \cdot \vec{J} = 0 \quad \text{------ (2)} \quad [\because \vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = 0]$$

But according to equation of continuity,

$$\vec{\nabla} \cdot \vec{J} = -\partial \rho / \partial t \quad \text{------ (3)}$$

Maxwell realized the situation and suggested that the total current density is incomplete & need something add to $\vec{J}$, such that eq^n (1) becomes,

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}' \quad ----- (4)$$

Now taking divergence of above eq^n

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot (\vec{J} + \vec{J}')$$

$$\vec{\nabla} \cdot \vec{J} = -\nabla \cdot J' \qquad [\because \vec{\nabla} \cdot \vec{J} = -\partial \rho / \partial t]$$

or $$\vec{\nabla} \cdot \vec{J}' = \partial \rho / \partial t$$

or $$\vec{\nabla} \cdot \vec{J}' = \frac{\partial}{\partial t}(\vec{\nabla} \cdot \vec{D}) \qquad [\because \underbrace{\vec{\nabla} \cdot \vec{D} = \rho}_{\text{(Maxwell first equation)}}]$$

$$\vec{\nabla} \cdot \vec{J}' = \vec{\nabla} \cdot \left(\frac{\partial}{\partial t}\vec{D}\right)$$

$$\therefore \boxed{J' = \frac{\partial \vec{D}}{\partial t}}$$

Therefore, eq^n (4) becomes,

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}} \rightarrow \text{Modified Ampere's law.}$$

Where $\frac{\partial \vec{D}}{\partial t}$ is displacement current density which arises due to changing electric field. In this way, displacement current leads to the modification of Ampere's law.

Ques: What is the difference between conduction current and displacement current? [2017-18]

Ans:

	Conduction Current	Displacement Current
1.	The electric current carried by conductors due to flow of charges is called conduction current.	1. The electric current due to changing electric field is called displacement current.
2.	It exist even if flow of electron is at uniform rate	2. It does not exist under steady condition
3.	$I_c = V/R$	3. $i_d = \varepsilon_0 \, d\phi_E / dt$

⇒ Maxwell Equations:

Ques: Derive Maxwell's equations in differential form. Give physical significance of each equation. [2017-18] Imp

Or

Write Maxwell's equation in integral and differential form and explain their physical significance with their proofs. [2018-19]

Ans: Maxwell Equations in Differential form:

(1) According to Gauss law in electrostatics:

$$\oint \vec{E}\cdot d\vec{S} = q/\varepsilon_0 = \frac{1}{\varepsilon_0}\int \rho\, dV$$

$$\oint \varepsilon_0 \vec{E}\cdot d\vec{S} = \int \rho\, dV$$

$$\oint \vec{D}\cdot d\vec{S} = \int \rho\, dV \quad ---(1) \quad [\because \vec{D} = \varepsilon_0 \vec{E}$$

Electric Displacement vector

Applying Gauss Divergence theorem,

$$\int (\vec{\nabla}\cdot\vec{D})\, dV = \int \rho\, dV$$

$$\int (\vec{\nabla}\cdot\vec{D} - \rho)\, dV = 0$$

for an arbitary volume, integrand must be zero,

$$\therefore \quad \vec{\nabla}\cdot\vec{D} - \rho = 0$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{D} = \rho} \longrightarrow \text{Maxwell's first Equation}$$

(2) According to Gauss law in magnetostatics,

$$\oint \vec{B}\cdot d\vec{S} = 0$$

Using Gauss Divergence theorem,

$$\oint \vec{B}\cdot d\vec{S} = \int (\vec{\nabla}\cdot\vec{B})\, dV = 0$$

for an arbitary volume, integrand should vanish for surface boundary, ∴ $\boxed{\vec{\nabla}\cdot\vec{B} = 0} \longrightarrow$ Maxwell's Second Equation

(3) According to Faraday's law of electromagnetic induction

$$emf = -d\phi_B/dt$$

Also, we know that $emf = \oint \vec{E}\cdot d\vec{l}$

$$\therefore \quad \oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s} \quad [\because \phi_B = \int \vec{B}\cdot d\vec{s}]$$

Magnetic flux

where 's' is surface bounded by circuit.

$$\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

$$\int (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} \quad \text{[Using Stokes theorem]}$$

$$\int \left(\vec{\nabla}\times\vec{E} + \frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{s} = 0$$

For an arbitary surface, the integrand should vanish, i.e.,

$$\vec{\nabla}\times\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}} \longrightarrow \text{Maxwell's third equation}$$

(4) According to Ampere's Circuital law,

$$\oint \vec{H}\cdot d\vec{l} = I$$

$$\oint \vec{H}\cdot d\vec{l} = \int \vec{J}\cdot d\vec{s} \quad [\because I = \int \vec{J}\cdot d\vec{s}]$$

$$\int (\vec{\nabla}\times\vec{H})\cdot d\vec{s} = \int \vec{J}\cdot d\vec{s} \quad \text{(using Stoke's theorem)}$$

$$\int (\vec{\nabla}\times\vec{H} - \vec{J})\cdot d\vec{s} = 0$$

For an arbitary surface, the integrand should vanish, i.e.,

$$\vec{\nabla}\times\vec{H} - \vec{J} = 0$$

$$\Rightarrow \vec{\nabla}\times\vec{H} = \vec{J} \quad \text{------- (2)}$$

$\hookrightarrow$ Conduction current density

The equation (2) derived on basis of ampere's law stands only for steady state current but for time varying fields the current density should be modified.

Take Divergence on both sides of equation (2)

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J}$$

or

$$\vec{\nabla}\cdot\vec{J} = 0 \quad ----(3) \quad [\because \vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = 0]$$

(Divergence of curl is zero) (DC)

But, by equation of continuity,

$$\vec{\nabla}\cdot\vec{J} = -\partial\rho/\partial t \quad ----(4)$$

Maxwell realized this contradiction and suggested that total current density in equation (2) is incomplete and suggested to add something to $\vec{J}$, such that equation (2) becomes

$$\vec{\nabla}\times\vec{H} = \vec{J} + \vec{J}' \quad ----(5)$$

Take divergence on both sides,

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \nabla\cdot(\vec{J}+\vec{J}')$$

$$\vec{\nabla}\cdot\vec{J} = -\vec{\nabla}\cdot\vec{J}' \quad [\because \vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = 0]$$

$$\vec{\nabla}\cdot\vec{J}' = \partial\rho/\partial t \quad [\because \vec{\nabla}\cdot\vec{J} = -\partial\rho/\partial t]$$

$$\vec{\nabla}\cdot\vec{J}' = \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D}) \quad [\because \vec{\nabla}\cdot\vec{D} = \rho]$$

$$\vec{\nabla}\cdot\vec{J}' = \nabla\cdot\left(\frac{\partial}{\partial t}\vec{D}\right)$$

$$\therefore \quad \vec{J}' = \frac{\partial}{\partial t}\vec{D}$$

Therefore, Equation (2)

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}} \rightarrow \text{Maxwell's fourth Equation}$$

The second term of above equation arises when electric displacement '$\vec{D}$' is changing with time and is called <u>Displacement current density</u>.

⊙ <u>Maxwell's Equation in Integral form:</u>

(1) We know that Maxwell's first equation in differential form is given as, $\vec{\nabla}\cdot\vec{D} = \rho$

Integrating over an entire volume,

$$\int(\vec{\nabla}\cdot\vec{D})\,dV = \int \rho\, dV$$

$$\oint \vec{D}\cdot d\vec{s} = \int \rho\, dV \quad \text{(Using Gauss Divergence theorem)}$$

$$\oint(\varepsilon_0\vec{E})\cdot d\vec{s} = \int \rho\, dV \quad (\because \vec{D} = \varepsilon_0\vec{E})$$

$$\boxed{\oint \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0}\int \rho\, dV} \rightarrow \text{Maxwell's first equation}$$

(2) Using Maxwell's second equation in differential form,

$$\vec{\nabla}\cdot\vec{B} = 0$$

Integrating over an entire volume, we get

This eq^n shows that monopole does not exist.

$$\int(\vec{\nabla}\cdot\vec{B})\,dV = 0$$

$$\boxed{\oint \vec{B}\cdot d\vec{s} = 0} \quad \text{(using Gauss Divergence theorem)}$$

→ Maxwell's second equation

(3) Maxwell's third equation in differential form,

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Integrating over an open surface area,

$$\int (\vec{\nabla} \times \vec{E}) \cdot d\vec{S} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \quad \text{(using stoke's theorem)}$$

$$\boxed{\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S}} \rightarrow \text{Maxwell's third equation}$$

(4) We know,

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Integrating over an open surface area, we get,

$$\int (\vec{\nabla} \times \vec{H}) \cdot d\vec{S} = \int \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$$

Applying Stoke's theorem,

$$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{S} + \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$$

$$\oint \vec{H} \cdot d\vec{l} = I + \int \frac{\partial}{\partial t} \vec{D} \cdot d\vec{S}$$

$$\boxed{\oint \vec{H} \cdot d\vec{l} = I + \frac{d}{dt} \int \vec{D} \cdot d\vec{S}} \rightarrow \text{Maxwell fourth Equation}$$

Physical Significance of Maxwell's Equations: V. Imp

(1) $\oint \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \int \rho \, dV$ → It is Gauss law in electrostatics. It states that the surface integral of Electric field over any closed surface area is equal to $\frac{1}{\epsilon_0}$ times of net charge enclosed by that surface.

(2) Show that magnetic monopoles does not exist. [20..]

$$\oint \vec{B}\cdot d\vec{s} = 0 \rightarrow \text{Gauss law in magnetostatics}$$

It states: ↳ net magnetic flux through any closed surface is zero.

↳ Magnetic monopoles does not exist.

↳ Also, signifies that magnetic field lines are closed curves.

(3) $$\underbrace{\oint \vec{E}\cdot d\vec{l}}_{\text{Induced emf around any closed path}} = \underbrace{-\frac{d}{dt}\int \vec{B}\cdot d\vec{s}}_{\text{negative rate of change of magnetic flux bounded by surface w.r.t. time}} \rightarrow \text{Faraday law of electromagnetic Induction}$$

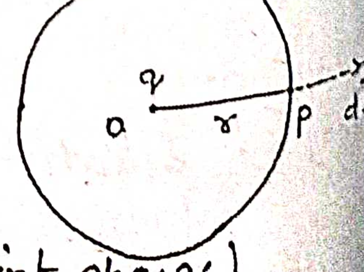
or ↳ Any changing magnetic field produces electric field.

(4) $$\oint \vec{H}\cdot d\vec{l} = I + \varepsilon \frac{d}{dt}\int \vec{E}\cdot d\vec{s} \rightarrow \text{Modified Ampere's law}$$

↳ Any current carrying conductor as well as time-varying electric field produces magnetic field.

Ques.: Deduce Coulomb's law of electrostatics from Maxwell's first equation. [2018–19]

Ans. Maxwell's first equation,



$$\oint \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0}\int \rho \, dV$$

or $$\oint \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0} \quad \text{------ (1)}$$

(q is point charge)

For positive point charge, $\vec{E} \rightarrow$ radially outward.

$\because$ $d\vec{s} \rightarrow$ also points out radially.

$\therefore$ $\vec{E}$ and $d\vec{s} \rightarrow$ parallel to each other.

$\therefore$ from equation (1), $\int E ds = q/\varepsilon_0$ [$\because \vec{E}\cdot d\vec{s} = E ds \cos\theta$ and $\theta = 0°$; $\theta \rightarrow$ angle between $\vec{E}$ and $d\vec{s}$]

$$E\int ds = q/\varepsilon_0$$

$$E = \frac{q}{4\pi r^2 \varepsilon_0} \Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Now at any point P, test charge 'q_0' is placed. Then, Electrostatic force experienced by 'q_0' is given by

$$\vec{F} = q_0\vec{E}$$

$$\boxed{\vec{F} = \frac{q_0 q}{4\pi\varepsilon_0 r^2}} \rightarrow \text{Coulomb's law}$$

Ques: Derive equation of continuity from Maxwell's fourth equation.

Ans: Maxwell' fourth equation, $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$

Taking divergence on both sides,

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)$$

$$\vec{\nabla}\cdot\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right) = 0 \qquad \left[\because \underbrace{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = 0}_{\text{(Divergence of curl (DC) = 0)}}\right]$$

$$\vec{\nabla}\cdot\vec{J} + \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D}) = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{J} + \frac{\partial}{\partial t}\rho = 0} \qquad \left[\because \underbrace{\vec{\nabla}\cdot\vec{D} = \rho}_{\text{Maxwell's first equation}}\right]$$

$\rightarrow$ Required Equation of continuity.

* Maxwell Equations in vaccum and in conducting medium.

As we know that general Maxwell's equations are

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \mu\left[\vec{J} + \frac{\partial\vec{D}}{\partial t}\right]$$

Here ρ and J are charge density and current density respectively.

$\epsilon \rightarrow$ permittivity

$\mu \rightarrow$ permeability of medium

where $\vec{D} = \epsilon\vec{E}$ (Displacement vector)

*** For free space or vaccum :-** There is no charge. i.e no conduction current so $\rho = 0$ and $\vec{J} = 0$

also $\epsilon = \epsilon_0$ and $\mu = \mu_0$

Therefore, Maxwell Equations in free space reduces to

- $\vec{\nabla}\cdot\vec{E} = 0$
- $\vec{\nabla}\cdot\vec{B} = 0$
- $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = -\mu_0\frac{\partial\vec{H}}{\partial t}$
- $\nabla\times\vec{B} = \mu_0\frac{\partial\vec{D}}{\partial t} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$ (as $\vec{D} = \epsilon_0\vec{E}$ for vaccum)

  or

  $\nabla\times\vec{H} = \epsilon_0\frac{\partial\vec{E}}{\partial t}$

*** Maxwell Equations for Conducting medium:-** In conducting medium, conduction current exists i.e ($\vec{J}\neq 0$) and charges are also present ($\rho\neq 0$). Now consider a uniform conducting linear medium having dielectric constant ϵ, permeability μ and conductivity σ. Now

$\vec{D} = \epsilon\vec{E}$ and $\vec{J} = \sigma\vec{E}$ in conducting medium.

So, Maxwell's equations reduces to.

- $\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon}}$ i.e $\vec{\nabla}\cdot\epsilon\vec{E} = \rho$ OR $\boxed{\vec{\nabla}\cdot\vec{D} = \rho}$

- $\boxed{\vec{\nabla}\cdot\vec{B} = 0}$

- $\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{OR} \quad \vec{\nabla}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}}$

- $\vec{\nabla}\times\vec{B} = \mu\left[\sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t}\right]$

  OR

  $\nabla\times\vec{H} = \sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t}$

⇒ Poynting Theorem.

Ques: State and deduce poynting theorem for the flow of e in an electromagnetic field. Also, discuss its physical signi V.I.

Or [2016-17, 2019-20, 2020-

Discuss the work-energy theorem for the flow of en in an electromagnetic field. [2016-17]

Or

What is poynting theorem? [2018-19]

Ans: Poynting theorem describes the flow of energy or po an electromagnetic field during propagation of uniform wave.

Derivation: Maxwell's third and fourth equation,

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad ---(1) \quad ; \quad \vec{\nabla}\times\vec{H} = \vec{J}+\frac{\partial\vec{D}}{\partial t} \quad ---$$

Taking dot product of (1) with $\vec{H}$ and eqn (2) with '$\vec{E}$',

$$\vec{H}\cdot(\vec{\nabla}\times\vec{E}) = -\vec{H}\cdot\left(\frac{\partial\vec{B}}{\partial t}\right) \quad ----(3)$$

$$\vec{E}\cdot(\vec{\nabla}\times\vec{H}) = \vec{E}\cdot\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right) \quad ----(4)$$

Subtract (4) from (3),

$$\vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{H}) = -\vec{H}\cdot\frac{\partial\vec{B}}{\partial t} - \vec{E}\cdot\vec{J} - \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}$$

⇓ vector identity

$$\left[\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{H})\right]$$

$$\therefore \nabla\cdot(\vec{E}\times\vec{H}) = -\vec{H}\cdot\frac{\partial\vec{B}}{\partial t} - \vec{E}\cdot\vec{J} - \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}$$

On rearranging,

$$-\nabla\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t} \quad ----(5)$$

Now, $\because$ $\vec{D} = \varepsilon \vec{E}$ and $\vec{B} = \mu \vec{H}$

$$\vec{E}\cdot\frac{\partial \vec{D}}{\partial t} = \varepsilon \vec{E}\cdot\frac{\partial \vec{E}}{\partial t} = \frac{1}{2}\varepsilon\frac{\partial}{\partial t}(\vec{E}\cdot\vec{E}) = \frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon E^2\right]$$

Similarly, $\vec{H}\cdot\frac{\partial \vec{B}}{\partial t} = \frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2\right]$

from eqn (5),

$$-\nabla\times(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon E^2\right] + \frac{\partial}{\partial t}\left[\frac{1}{2}\mu B^2\right]$$

taking volume integral of above equation

$$-\int_V \vec{\nabla}\cdot(\vec{E}\times\vec{H})\,dV = \int_V (\vec{E}\cdot\vec{J})\,dV + \frac{d}{dt}\int_V \frac{1}{2}\left[\varepsilon E^2 + \mu H^2\right] dV$$

Applying Gauss divergence theorem on LHS, we get

$$\boxed{\underbrace{\int_V (\vec{E}\cdot\vec{J})\,dV}_{\mathrm{I}} = \underbrace{-\frac{d}{dt}\int_V \frac{1}{2}\left[\varepsilon E^2 + \mu H^2\right] dV}_{\mathrm{II}} \underbrace{- \oint_S (\vec{E}\times\vec{H})\cdot d\vec{S}}_{\mathrm{III}}}$$

$\hookrightarrow$ Represent work energy theorem and is called poynting theorem.

Physical Significance

- I term represents the total power dissipated in a volume V.
- II term represents the sum of energy stored in electric field $\left(\frac{\varepsilon}{2}\int E^2 dV\right)$ and in magnetic field $\left(\frac{1}{2}\mu^2\int H^2 dV\right)$.
- III term represents the rate at which the energy is carried out of volume V, across its boundary surface by electromagnetic wave.

or we can say that the physical significance of poynting theorem is "Conservation of energy", as it is clear from eqn of poynting theorem that the work done on the charge by an EM force is equal to decrease in energy stored in field, less than the energy which flowed out through surface.

⇒ Poynting Vector:

Imp

Ques: What is poynting vector? [2016-17, 2018-19]

Ans: The amount of field energy passing through unit area of the surface per unit time is called Poynting Vector. It is represented by

$$\boxed{\vec{S} = \vec{E} \times \vec{H}}$$

∵ $\vec{E}$ and $\vec{H}$ are perpendicular to each other, the magnitude of $\vec{S}$ is given as,

$$\boxed{S = EH}$$

[∵ $\vec{S} = \vec{E} \times \vec{H}$
$S = EH \sin 90$
$= EH$]

→ It is along the direction of wave propagation.

Ques:- What is unit and dimension of poynting vector?

Ans: Unit : Joule/m^2.sec or Watt/m^2

Dimension: $[S] = \dfrac{\text{Energy}}{\text{Area} \cdot \text{Time}} = \dfrac{ML^2T^{-2}}{L^2 T}$

∴ Dimension of 'S' is MT^{-3}

⇒ Electromagnetic wave in free space (Vacuum) V. Imp

Ques: Deduce four Maxwell's equations in free space. [2019-20, 2020-21]

Maxwell's equations are :

$$\vec{\nabla}\cdot\vec{D} = \rho \quad \text{-------} (1')$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{-----} (2')$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{-----} (3')$$

$$\vec{\nabla}\times\vec{H} = J + \frac{\partial \vec{D}}{\partial t} \quad \text{----} (4')$$

Vacuum means that there is no free charge in the region so there is no current produced. So, we have

$$\boxed{\rho = 0,\ \vec{J} = 0,\ \vec{D} = \varepsilon_0\vec{E},\ \vec{B} = \mu_0\vec{H}}$$

∴ Maxwell's equation in free space.

$$\boxed{\begin{aligned} \vec{\nabla}\cdot\vec{E} &= 0 \quad \text{-----} (5') \\ \vec{\nabla}\cdot\vec{B} &= 0 \quad \text{-----} (6') \\ \vec{\nabla}\times\vec{E} &= -\mu_0\frac{\partial \vec{H}}{\partial t} \quad \text{-----} (7') \\ \vec{\nabla}\times\vec{H} &= \varepsilon_0\frac{\partial \vec{E}}{\partial t} \quad \text{-----} (8') \end{aligned}}$$

Ques: Derive the electromagnetic wave equations in free space. Prove that the electromagnetic wave propogate with speed of light in free space. [2017-18] V. Imp

Or

Derive the equation for propogation of plane electromagnetic wave in free space. Show that the velocity of plane electro-magnetic wave in free space is given by $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$.

[2015-16, 2018-19]

Ans: Maxwell's equation in free space:

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \text{------} \quad (1.)$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{------} \quad (2.)$$

$$\vec{\nabla}\times\vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad \text{------} \quad (3.)$$

$$\vec{\nabla}\times\vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{------} \quad (4.)$$

Taking curl of equation (3.),

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\vec{\nabla}\times\left(\mu_0 \frac{\partial \vec{H}}{\partial t}\right)$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu_0 \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H})$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu_0 \frac{\partial}{\partial t}\left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \quad \text{(Using eq}^n\text{ (4))} \Rightarrow$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Vector identity, $\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \underbrace{\vec{\nabla}(\vec{\nabla}\cdot\vec{E})}_{\text{zero } (\because \vec{\nabla}\cdot\vec{E}=0)} - \nabla^2\vec{E}$

$$\therefore \quad -\nabla^2\vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

or

$$\boxed{\nabla^2\vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \quad \text{------} \quad (5.)$$

Similarly,

$$\boxed{\nabla^2\vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}} \quad \text{------} \quad (6.)$$

Required wave equ for the propagation of EM waves free space

Generally, wave equation as,

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \quad \text{------} \quad (7.)$$

; where ψ - wave fu propagates with velocity 'v'.

Compare eq (7.) with (5.) & (6.), we observe that $\vec{E}$ and $\vec{H}$ propagates with the velocity of $\boxed{v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}}$ in free space.

$$\therefore \ v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$= \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}}$$

$$= 2.99 \times 10^{8}\ m/s = c \ (\text{speed of light})$$

Thus, EM waves travel in free space with speed of light.

$\therefore$ Eq. (5') & (6') can be written as,

$$\boxed{\begin{aligned} \nabla^2 \vec{E} &= \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} \\ \nabla^2 \vec{H} &= \frac{1}{c^2}\frac{\partial^2 \vec{H}}{\partial t^2} \end{aligned}}$$

⇒ Electromagnetic wave in non-conducting media:

Ques: Write the Maxwell's equation in non conducting medium. Also, derive electromagnetic wave equation in non-conducting medium.

Ans: Non-conducting media means non-charged, current-free dielectric. Therefore, we have conditions

$$\rho = 0, \ \vec{J} = 0, \ \vec{D} = \varepsilon \vec{E}, \ \vec{B} = \mu \vec{H}$$

Use the above condition in eq. (1'), (2'), (3') and (4'). So, we have Maxwell's equation in non-conducting media as

$$\boxed{\begin{aligned} \vec{\nabla}\cdot\vec{E} &= 0 \\ \vec{\nabla}\cdot\vec{B} &= 0 \\ \vec{\nabla}\times\vec{E} &= -\mu\frac{\partial \vec{H}}{\partial t} \\ \vec{\nabla}\times\vec{H} &= \varepsilon\frac{\partial \vec{E}}{\partial t} \end{aligned}}
\begin{aligned} &\text{---(1)} \\ &\text{---(2)} \\ &\text{---(3)} \\ &\text{---(4)} \end{aligned}$$

Taking curl of eq. (3),

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu \ \nabla\times\left(\frac{\partial \vec{H}}{\partial t}\right)$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H})$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$ [Using eq (1.4)]

Using vector identity,

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \underbrace{\vec{\nabla}(\vec{\nabla}\cdot\vec{E})}_{\text{zero } (\because\ \vec{\nabla}\cdot\vec{E}=0)} - \nabla^2\vec{E}$$

$\therefore$

$$\boxed{\begin{aligned}\nabla^2\vec{E} &= \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}\\ \nabla^2\vec{H} &= \mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2}\end{aligned}}$$

Similarly → Required electromagnetic wave equation in non-conducting media

Ques: Prove that electromagnetic waves are transverse in nature. [2017-18, 2018-19] V. Imp

Or

Show that electric & magnetic vectors are normal to the direction of propagation of electromagnetic wave. [2020-21]

Or

Show that $\vec{E}$, $\vec{H}$ and direction of propagation form a set of orthogonal vectors. [2016-17]

Ans: The wave equation for electric and magnetic field in free space are:

$$\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \quad ; \quad \nabla^2\vec{H} = \mu_0\varepsilon_0\frac{\partial^2 H}{\partial t^2}$$

Solution of above equations written as,

$$\vec{E}(r,t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{H}(r,t) = \vec{H}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

Where E_0 & H_0 are amplitudes and 'k' is wave propagation vector, given as

$$\vec{K} = \underbrace{K}_{\text{magnitude of } \vec{K} \text{ vector}}\hat{n} = \frac{2\pi}{\lambda}\hat{n} = \frac{2\pi\nu}{c}\hat{n} = \frac{\omega}{c}\hat{n}$$

Here, '$\hat{n}$' → unit vector in direction of wave propagation.

$\because$ There is no angular coordinates in $\vec{E}$ and $\vec{H}$,

$\therefore$ $\nabla = \partial/\partial r$

Now, $\frac{\partial}{\partial r}\vec{E}(r,t) = \frac{\partial}{\partial r}\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$= ik\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ $\qquad \left[\because \frac{\partial}{\partial x}e^{ix} = ie^{ix}\right]$

$\frac{\partial}{\partial r}\vec{E}(r,t) = ik\vec{E}(r,t)$

$\Rightarrow \boxed{\frac{\partial}{\partial r} = ik = \nabla}$ ------- (1)

Now, $\frac{\partial}{\partial t}\vec{E}(r,t) = \frac{\partial}{\partial t}\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$= -i\omega\vec{E}(r,t)$

$\Rightarrow \boxed{\frac{\partial}{\partial t} = -i\omega}$ ——— (2)

Using eq^n (1) in eq^n $\vec{\nabla}\cdot\vec{E} = 0$

$\therefore i\vec{k}\cdot\vec{E} = 0$

$\vec{k}\cdot\vec{E} = 0$ $\qquad [\because \vec{k}\cdot\vec{E} = |\vec{k}||\vec{E}|\cos\theta]$

$\Rightarrow \boxed{\vec{k}\perp\vec{E}}$ ——— (3)

Similarly, Using eq^n (1) in eq^n $\vec{\nabla}\cdot\vec{H} = 0$

$\therefore i\vec{k}\cdot\vec{H} = 0$

$\vec{k}\cdot\vec{H} = 0$

$\boxed{\vec{k}\perp\vec{H}}$ ——— (4)

Now, substitute the value of ∇ and $\partial/\partial t$ in eqn $\vec{\nabla}\times\vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$,

$$\therefore \quad i(\vec{k}\times\vec{E}) = \mu_0 i\omega\vec{H}$$

$$k(\hat{n}\times\vec{E}) = \mu_0\omega\vec{H}$$

$$\frac{k}{\mu_0\omega}(\hat{n}\times\vec{E}) = \vec{H}$$

$$\boxed{\frac{1}{\mu_0 c}(\hat{n}\times\vec{E}) = \vec{H}} \qquad \left[\because \frac{\omega}{k} = c\right]$$

From above eqn, it is clear that field vector $\vec{H}$ perpendicular to both $\vec{k}$ and $\vec{E}$.

Thus, we see that electric field $\vec{E}$, magnetic $\vec{H}$ and direction of propagation $\vec{k}$ are mutually perpendicular to each other, i.e, EM waves are transverse in nature.

⇒ Relation between $\vec{E}$ and $\vec{H}$:

Ques: What do you mean by impedance of a wave? [2019-20]

or

What do you understand by characteristics impedance?

Ans. The ratio of magnitude of $\vec{E}$ to the magnitude of $\vec{H}$ is known as characteristic impedance, denoted by 'Z_0'.

Using value of $\partial/\partial r$ & $\partial/\partial t$ from eq. (1) & (2) in eq^n $\vec{\nabla}\times\vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$, we get.

$$i(\vec{K}\times\vec{E}) = +\mu_0 i\omega\vec{H}$$

$$\vec{K}\times\vec{E} = \mu_0\omega\vec{H}$$

$$|\vec{K}||\vec{E}|\sin\theta = \mu_0\omega|\vec{H}| \quad [\theta \to \text{angle between } \vec{K} \,\&\, \vec{E}]$$

$$KE = \mu_0\omega H \quad [\because \theta = 90^\circ]$$

$$E = \mu_0\frac{\omega}{K}H$$

or $$\boxed{E = \mu_0 c H} \quad \left[\because K = \frac{\omega}{c} \text{ or } \frac{\omega}{v}\right]$$

$$E = \mu_0\frac{1}{(\mu_0\varepsilon_0)^{1/2}}H \quad \left[\because c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]$$

$$\boxed{E = \sqrt{\frac{\mu_0}{\varepsilon_0}}\,H}$$

where $\mu_0 = 4\pi\times10^{-7}\ N/A^2$
$\varepsilon_0 = 8.854\times10^{-12}\ m^{-2}kg^{-1}s^{4}A^{2}$

$$\therefore \boxed{Z_0 = \left|\frac{\vec{E}}{\vec{H}}\right| = \left|\frac{E_0}{H_0}\right| = \mu_0 c = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.72\ \text{ohms}}$$

↳ Dimensions of electric constant.
↳ It is universal constant.

* Relation between Electric and magnetic fields of Electromagnetic wave.

Note:- The ratio of Electric field to magnetic field of e.m wave is always equal to speed of propagation of wave in that medium.

Since, E.M wave have sinusoidally varying electric and magnetic fields which are given as.

$$E = E_0 \cos(kx - \omega t) \quad -(1)$$

$$B = B_0 \cos(kx - \omega t) \quad -(2)$$

If λ be the wavelength of e.m wave and ω is angular frequency then

$$\frac{\omega}{k} = \frac{2\pi f}{\left(\frac{2\pi}{\lambda}\right)} = f\lambda = v$$

Where v is velocity of propagation of wave, for free space it is equal to speed of light i.e $v = c$.

taking partial differentiation of eq(1) and eq(2), We get

$$\frac{\partial E}{\partial x} = -k E_0 \sin(kx - \omega t) \quad -(3)$$

$$\frac{\partial B}{\partial t} = -\omega B_0 \sin(kx - \omega t) \quad -(4)$$

According Faraday's law $\left(\nabla \times \vec{E} = -\frac{\partial B}{\partial t} \right)$

i.e $$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$-k E_0 \sin(kx - \omega t) = -\omega B_0 \sin(kx - \omega t)$$

$$KE_0 = \omega B_0$$

$$\frac{E_0}{B_0} = \frac{\omega}{K}$$

$$\boxed{\frac{E_0}{B_0} = |\vec{v}|}$$ Hence Proved

For free space $(v = c)$

$$\boxed{\frac{E_0}{B_0} = c}$$

Relation b/w Electric field and magnetic field (magnetic flux density) of e.m wave

Also, We Know that $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ and $v = \frac{1}{\sqrt{\mu \epsilon}}$

Similarly $\vec{B_0} = \mu_0 \vec{H_0}$ (for free space) and $\vec{B_0} = \mu \vec{H_0}$ (For dielectric media)

So, Relation between electric field $\vec{E}$ and magnetic field intensity $\vec{H}$ is

For Free space, $\frac{E_0}{B_0} = c$

$$\frac{E_0}{H_0} = \frac{\mu_0 \times 1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\epsilon_0}}$$ (characteristic Impedance)

For dielectric media. $\frac{E_0}{B_0} = v$.

$$\frac{E_0}{\mu H_0} = \frac{1}{\sqrt{\mu \epsilon}}$$

$$\boxed{\frac{E_0}{H_0} = \sqrt{\frac{\mu}{\epsilon}}}$$ (characteristic Impedance for dielectric media).

* Plane Electromagnetic waves in conducting medium:-

We know that, Maxwell's equations in conducting medium is given by

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon} \quad —(1)$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad —(2)$$

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} \quad —(3)$$

$$\vec{\nabla}\times\vec{H} = \sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t} \quad —(4)$$

Now, taking curl of eq(3) on both side.

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu\frac{\partial}{\partial t}(\nabla\times\vec{H})$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}\left(\sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t}\right)$$

[From eq (1) $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$ but we know that there no net charge within a conductor because the charge resides on the surface of the conductor, $\rho = 0$ so $\vec{\nabla}\cdot\vec{E} = 0$]

Now, $$0 - \nabla^2\vec{E} = -\mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

i.e $$\nabla^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

** $$\boxed{\nabla^2\vec{E} - \mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0}$$ ← wave equa for conducti media in te of Electric

Now, taking curl of both sides of eq(4)

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \sigma(\vec{\nabla}\times\vec{E}) + \epsilon\frac{\partial}{\partial t}(\nabla\times\vec{E})$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{H}) - \nabla^2\vec{H} = -\sigma\mu\frac{\partial\vec{H}}{\partial t} - \mu\epsilon\frac{\partial^2 H}{\partial t^2}$$ {usi $\nabla\times$ =

As $\nabla \cdot \vec{H} = 0$

We get
$$0 - \nabla^2 \vec{H} = -\sigma \mu \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 H}{\partial t^2}$$

i.e
$$-\nabla^2 H = -\sigma \mu \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 H}{\partial t^2}$$

i.e **
$$\boxed{\nabla^2 H - \sigma \mu \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 H}{\partial t^2} = 0}$$
← wave equation for conducting media in terms of magnetic field.

* The solution of the wave equation for conducting media is of the form.

$$\vec{E} = \vec{E_0} e^{-\gamma z}$$

where $\gamma = \alpha + i\beta$

$\alpha \rightarrow$ real part and is called as attenuation constant

$\beta \rightarrow$ imaginary part and is called as phase constant.

Value of α is

$$\boxed{\alpha = \omega \sqrt{\left(\frac{\mu \epsilon}{2}\right)\left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} - 1\right]}}$$

and

$$\boxed{\beta = \omega \sqrt{\left(\frac{\mu \epsilon}{2}\right)\left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} + 1\right]}}$$

⇒ Depth of penetration (Skin depth):

Ques: What do you mean by depth of penetration? Imp [2016-17, 2018-19]

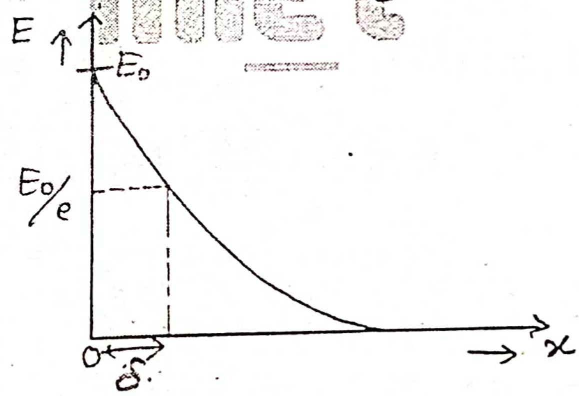
Ans: When an EM wave propagate in a medium, its amplitude decreases inside the medium from the surface. This phenomenon is known as attenuation.

The amplitude of an EM wave at a depth 'x' inside the medium is given by $\boxed{E = E_0 e^{-\alpha x}}$

where $E_0 \rightarrow$ amplitude of wave at the surface of medium

$\alpha \rightarrow$ attenuation constant.

The depth of penetration is defined as the depth in which the strength of electric field associated with the electromagnetic wave reduces to $\frac{1}{e}$ times to its initial value.



Ques: What is the relation between skin depth and attenuation constant?

Ans. $\because \quad E = E_0 e^{-\alpha x}$

At skin depth $x = \delta$, $E = E_0/e$

$$\therefore \quad \frac{E_0}{e} = E_0 e^{-\alpha\delta}$$

$$e^{-1} = e^{-\alpha\delta} \quad \Rightarrow \quad \alpha\delta = 1$$

$$\Rightarrow \boxed{\delta = \frac{1}{\alpha}} \rightarrow$$ Required relation between skin depth and attenuation constant.

$\Rightarrow$ skin depth is inversely propotional to attenuation constant

Ques: Derive an expression of skin depth for good conductor and insulator. Also, show that skin depth for insulator does not depends on frequency of EM wave.

Imp

Ans: Attenuation constant 'α' is given as,

$$\alpha = \omega\left[\frac{\mu\varepsilon}{2}\left\{\left(1+\frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{1/2}-1\right\}\right]^{1/2} \text{---} (1)$$

for good conductor,

$$\boxed{\frac{\sigma}{\omega\varepsilon} >> 1}$$

$\therefore$ Neglect 1 with respect to $\frac{\sigma}{\omega\varepsilon}$ in eq^n (1)

$$\alpha = \omega\left\{\frac{\mu\varepsilon}{2}\left(\frac{\sigma}{\omega\varepsilon}-1\right)\right\}^{1/2}$$

$$\alpha = \omega\left\{\frac{\mu\varepsilon}{2}\frac{\sigma}{\omega\varepsilon}\right\}^{1/2}$$

$$\alpha = \sqrt{\frac{\mu\sigma\omega}{2}}$$

$\therefore$ Skin depth, $\boxed{\delta = \sqrt{\frac{2}{\mu\sigma\omega}}} \rightarrow$ Expression of skin depth for good conductor

From above equation, it is clear that skin depth for good conductor depends on frequency of EM wave.

Above eqⁿ can also be written as.

$$\boxed{\delta = \sqrt{\frac{1}{\mu\sigma\pi f}}}$$

[∵ $\omega = 2\pi f$] angular frequency

⇒ **For Insulators:**

$$\boxed{\frac{\sigma}{\omega\varepsilon} << 1}$$

$$\therefore \left(1+\frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{1/2} \approx 1 + \frac{\sigma^2}{2\omega^2\varepsilon^2}$$

Now, eqⁿ (iii) can be written as,

$$\alpha = \omega\left\{\frac{\mu\varepsilon}{2}\left(1+\frac{\sigma^2}{2\omega^2\varepsilon^2}-1\right)\right\}^{1/2} = \omega\left(\frac{\mu\varepsilon}{2}\frac{\sigma^2}{2\omega^2\varepsilon^2}\right)^{1/2}$$

$$\alpha = \frac{\sigma}{2}\left(\frac{\mu}{\varepsilon}\right)^{1/2}$$

$$\therefore \boxed{\delta = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}}$$ → Required expression of skin depth for insulators

It is clear from above equation that skin depth for insulators does not depend upon frequency of EM wave.

Ques: For a conducting medium, $\sigma = 5.8 \times 10^6$ siemens/m and $\varepsilon_r = 1$. Find out conduction and displacement current densities if magnitude of electric field intensity E is given by $E = 150 \sin(10^{10} t)$ Volt/m. [2018-19]

Solⁿ: Conduction current density, $\vec{J}_c = \sigma\vec{E}$

$$\vec{J}_c = 5.8 \times 10^6 \times 150 \sin(10^{10} t)$$

$$\vec{J}_c = 8.7 \times 10^8 \sin(10^{10} t)\ A/m^2$$

Displacement current density, $\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t}$

$$\vec{J}_d = \varepsilon_0 \varepsilon_r \frac{\partial \vec{E}}{\partial t}$$

$$= 8.854 \times 10^{-12} \times 1 \times \frac{\partial}{\partial t}[150 \sin(10^{10} t)]$$

$$= 8.854 \times 10^{-12} \times 150 \times 10^{10} \cos(10^{10} t)$$

$$\vec{J}_d = 13.28 \cos(10^{10} t)\ A/m^2 \quad \text{Ans}$$

Ques:- Calculate the magnitude of poynting vector at surface of the sun. Given that power radiated by Sun is 5.4×10^{28} watt and its radius is 7×10^8 m. [2018-19]

Solⁿ: Poynting vector, $S = \frac{\text{Power radiated}}{4\pi R^2}$

$$= \frac{5.4 \times 10^{28}}{4 \times 3.14 \times (7 \times 10^8)^2}$$

$$\boxed{S = 8.77 \times 10^9\ watt/m^2} \quad \text{Ans:}$$

Ques: If the magnitude of H in a plane wave is 1 amp/meter, find the magnitude of E for plane wave in free space. [2015-16] Imp

Solⁿ: We know that characteristic impedance is given by

$$\frac{H_0}{E_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}}$$

where E_0 & H_0 are the magnitude of $\vec{E}$ and $\vec{H}$.

$$\therefore E_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} H_0 = 1 \times \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} \Rightarrow \boxed{E_0 = 376.72\ V/m}$$

Ans.

Ques: A 100-watt sodium lamp radiating its power. Calculate the electric field and magnetic field strength at a distance of 5m from the lamp. [2015-16; 2018-19] V. Imp

Sol^n, We know that Poynting vector is given by

$$S = \frac{P}{4\pi r^2}$$

$$= \frac{100\,W}{4 \times 3.14 \times (5m)^2}$$

$$S = 3.185 \times 10^{-1} \frac{J}{m^2 s}$$

$$\because \; S = \vec{E} \times \vec{H} = EH \sin 90 = EH$$

$$\therefore \; EH = 3.185 \times 10^{-1}\, J/m^2 s \quad \text{------ (1)}$$

We also know that,
$$\frac{E}{H} = 376.6\,\Omega \quad \text{---- (2)}$$

Using (1) and (2), we get

$$E^2 = 119.95 \frac{V^2}{m^2}$$

or $\boxed{E = 10.95\, V/m}$

Now, from eq^n (2),

$$H = \frac{E}{376.6} = \frac{10.95}{376.6}$$

$$\boxed{H = 0.0291 \text{ amp-turn}/m}$$

Ques: The sunlight strikes the upper atmosphere of earth with energy flux 1.38 kW/m^2. What will be the peak values of electric & magnetic field at the points? [2019-20]

Sol^n: Energy flux, $S = 1.38\, kW/m^2$

$$= 1380\, W/m^2$$

$\because \quad S = \vec{E} \times \vec{H} = EH \sin 90^\circ = EH$

$\therefore \quad EH = 1380 \; W/m^2 \text{ or } 1380 \frac{J}{m^2 s}$ ---- (1)

Also, we know that,

$$\frac{E}{H} = 376.6 \; \Omega \quad \text{---} (2)$$

From eqn (1) & (2),

$$E^2 = 1380 \times 376.6$$
$$E^2 = 519708 \; V^2/m^2$$
$$\boxed{E = 720.91 \; V/m}$$

$$\because \quad E = E_0/\sqrt{2}$$

$$\therefore \quad E_0 = E\sqrt{2} = 720.91 \times 1.414$$

$$\boxed{E_0 = 1.02 \; kV/m}$$

Put value of E, in Eqn (1),

$$H = \frac{1380}{720.91} = 1.91 \; \frac{\text{amp-turn}}{m}$$

$$\therefore \quad H_0 = 1.91 \times \sqrt{2} = 1.91 \times 1.414$$

$$\boxed{H_0 = 2.7 \; \frac{\text{amp-turn}}{m}}$$

Ques: For silver, $\mu = \mu_0$ and $\sigma = 3 \times 10^7$ mhos/m. Calculate the skin depth at 10^8 Hz frequency. [Given $\mu_0 = 4\pi \times 10^{-7} \; N/A^2$] [2016-17]

Soln: Skin depth for good conductor is given by

$$\delta = \sqrt{\frac{2}{\mu \sigma \omega}} = \sqrt{\frac{2}{\mu \sigma 2\pi f}} = \sqrt{\frac{1}{\mu \sigma \pi f}}$$

$$\delta = \sqrt{\frac{1}{4\pi \times 10^{-7} \times 3 \times 10^7 \times 3.14 \times 10^8}} = 9.18 \times 10^{-6} \; m \quad \text{Ans}$$

Ques: Using Maxwell's equation, curl $\vec{B} = \mu_0\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$, prove div $\vec{D} = \rho$.

Soln. According to given problem,

$$\text{curl } \vec{B} = \mu_0\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

Taking divergence of both sides, we get

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0 \vec{\nabla}\cdot\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

$$\vec{\nabla}\cdot\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) = 0 \qquad [\because \vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}$$

$$\vec{\nabla}\cdot\vec{J} + \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D}) = 0$$

From eqn of continuity,

$$\vec{\nabla}\cdot\vec{J} = -\partial\rho/\partial t$$

Hence,

$$\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D}) = \frac{\partial \rho}{\partial t}$$

$$\boxed{\vec{\nabla}\cdot\vec{D} = \rho}$$

Ques: If the earth receives $2\,Cal\,min^{-1}\,cm^{-2}$ solar energy. What are the amplitudes of electric and magnetic fields of radiation. V.Imp

Soln. According to Poynting Vector,

$$\vec{S} = \vec{E}\times\vec{H}$$

Energy flux per unit area per second

$$|\vec{S}| = |\vec{E}\times\vec{H}| = EH\sin 90° = EH$$

$$|S| = \frac{2\,Cal}{min\,cm^2} = \frac{2\times 4.2\,J}{60\,sec\times 10^{-4}\,cm^2} \qquad \because 1\,Cal = 4.2\,J,\ 1\,min = 60\,sec,\ 1\,cm = 10^{-2}\,m$$

$$S = 1400 = EH \quad \text{—(1)}$$

We know from characteristic Impedance $S = \frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.7$

multiply eq (1) & (2)

$$EH\times\frac{E}{H} = 1400\times 376.72$$

$$E^2 = 5.274\times 10^5 \Rightarrow E = 726.2\,V/m$$

Substituting value of E in eq (1) $EH = 1400 \Rightarrow 726.2\times H = 1400$

$$\Rightarrow H = \frac{1400}{720.2} = 1.928\ \text{Amp. turn/m}$$

Amplitude of Electric & magnetic field $E_0 = \frac{E}{\sqrt{2}} \Rightarrow E = E_0\sqrt{2} = 726 \ldots = 10 \ldots$

& $H = H_0\sqrt{2} = 1.928\sqrt{2} \ldots$

5 Year's

University Paper Questions

(AKTU Question Bank)

B. Tech I Year [Subject Name: Engineering Physics]

5 Years AKTU University Examination Questions		Unit-2	
S. No	Questions	Session	Lecture No
1	Derive a suitable expression for continuity equation. Give its physical significance. Or What is the equation of continuity? Obtain the required expression for it. Also give its physical significance.	2016-17 2018-19	10
2	What is displacement current?	2015-16 2016-17 2018-19 2020-21	10
3	What is the difference between conduction current and displacement current?	2017-18	10
4	Explain the concept of displacement current and show how it leads to modification of Ampere law.	2016-17 2018-19 2019-20	10
5	Why Maxwell proposed that Ampere law require modification?	2018-19	10
6	Derive Maxwell's equations in differential form. Give physical significance of each equation. or Write Maxwell's equations in integral and differential form and explain their physical significance with their proofs.	2017-18 2018-19	11
7	Deduce Coulomb's law of electrostatics from Maxwell's first equation.	2018-19	11
8	Show that magnetic monopoles do not exist.	2020-21	11
9	Deduce four Maxwell's equations in free space.	2019-20 2020-21	12
10	What is Poynting theorem? or State and deduce Poynting theorem for the flow of energy in an electromagnetic field. or Discuss the work-energy theorem for the flow of energy in an electromagnetic field.	2016-17 2018-19 2019-20 2020-21	13
11	Discuss the physical significance of Poynting theorem.	2020-21 2016-17	13
12	What is Poynting vector?	2016-17 2018-19	13

13	Derive the electromagnetic wave equations in free space. Prove that the electromagnetic waves propagate with speed of light in free space. or Derive the equation for the propagation of plane electromagnetic wave in free space. Show that the velocity of plane electromagnetic wave in free space is given by $c = 1/\sqrt{\mu_0 \epsilon_0}$.	2015-16 2017-18 2018-19 2020-21 2021-22	14
14	Prove that electromagnetic waves are transverse in nature. or Show that electric and magnetic vectors are normal to the direction of propagation of electromagnetic wave. or Show that E, H and direction of propagation form a set of orthogonal vectors.	2016-17 2017-18 2018-19 2020-21 2021-22	14
15	What do you mean by impedance of a wave?	2019-20	14
16	What do you mean by depth of penetration or skin depth?	2016-17 2018-19	15
17	Define the concept of skin depth for high and low frequency waveforms.	2021-22	16
18	For a conducting medium, $\sigma = 5.8 \times 10^6$ Siemens/m and $\epsilon_r = 1$. Find out the conduction and displacement current densities if the magnitude of electric field intensity E is given by $E = 150 \sin(10^{10} t)$ Volt/m.	2018-19 2020-21	16
19	Calculate the magnitude of Poynting vector at the surface of the Sun. Given that power radiated by Sun is 5.4×10^{28} W and its radius is 7×10^8 m.	2018-19	16
20	If the magnitude of H in a plane wave is 1 amp/meter, find the magnitude of E for plane wave in free space.	2015-16	16
21	In an electromagnetic wave, the electric and magnetic fields are 100 V/m and 0.265 A/m. What is maximum energy flow?	2021-22	16
22	A 100-watt sodium lamp radiating its power. Calculate the electric field and magnetic field strength at a distance of 5m from the lamp.	2015-16 2018-19	16
23	Assuming that all the energy from a 1000 watt lamp is radiated Uniformly. Calculate the average values of the intensities of electric and Magnetic fields of radiation at a distance of 2 m from the lamp.	2021-22	16
24	The sunlight strikes the upper atmosphere of earth with energy flux $1.38 kWm^{-2}$. What will be the peak values of electric and magnetic field at the points?	2019-20	16
25	For silver, $\mu = \mu_o$ and $\sigma = 3 \times 10^7$ mhos/m. Calculate the skin depth at 10^8 Hz frequency. [Given, $\mu_o = 4\pi \times 10^{-7}$ N/A^2]	2016-17	16
26	If the earth receives 2 cal /(min-cm square) solar energy, what are the amplitudes of electric and magnetic fields of radiation?	2009-10	16