Regular Course Handbook

Physics Group

Unit-1 (Extra Notes: As Per New Syllabus)

B.Tech I Year

B.Tech I Year

Regular Course Handbook

Subject Name: Engineering Physics (Unit-1)

UNIT - I : QUANTUM MECHANICS

Inadequacy (limitation) of classical mechanics →

The classical mechanics explains correctly motion of celestial bodies like planet, star and macroscopic bodies. The Inadequacies of classical mechanics are given below

1. It does not hold in region of Atomic dimension.
2. It could not explain stability of atoms.
3. It could not explain observed spectrum of black body radiation.
4. It could not explain observed variation of specific heat of metals & gases.
5. It could not explain the origin of discrete spectra of atoms
6. classical mechanics also could not explain large number of observed phenomena like photoelectric effect, Compton effect, Raman effect Anomalous Zeeman effect.

The Inadequancy of classical mechanics leads to the development of Quantum mechanics

UNIT I : QUANTUM MECHANICS

⇒ Introduction:

Quantum mechanics is a physical science dealing with the behaviour of matter and energy on the scale of atoms and subatomic particles/waves.

The term quantum mechanics was first coined by Max Born in 1924.

Ques: What do you mean by black body?

Ans: A perfectly black body is that which absorbs completely the radiations of all wavelengths incident on it.

A perfectly black body cannot be realized in practice, but materials like Platinum black or Lamp black come close to being ideal black bodies. Such materials absorbs 96% to 98% of incident radiations.

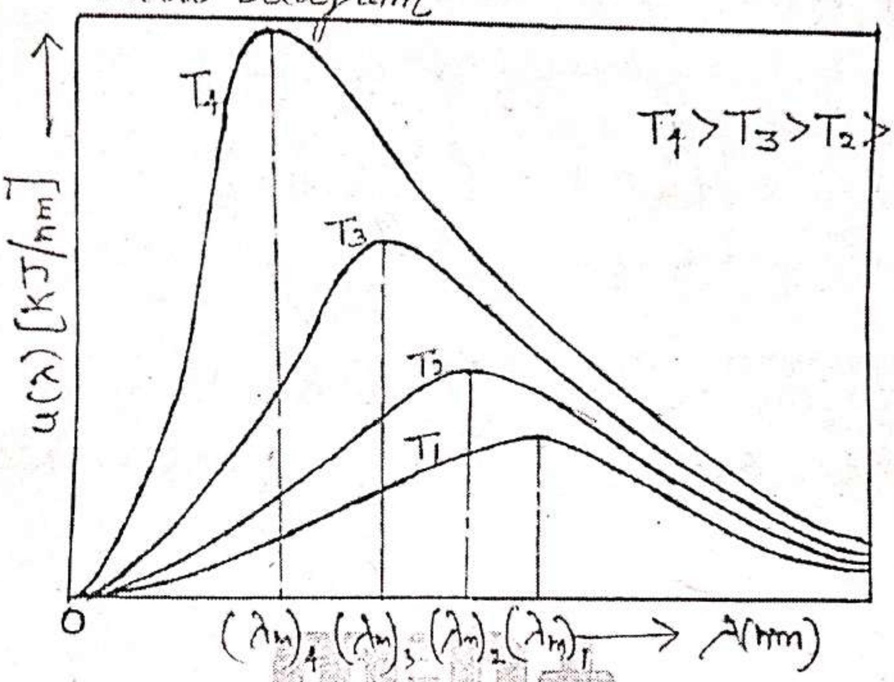
Example: Nearest example of an ideal black body is Ferry's black body.

Ques:- Describe energy distribution in black body radiation? [2016-17]

Or

Explain black body spectrum with proper diagram.

Ans: Black body spectrum :- Lummer and Pringsheim investigated the distribution of energy amongst different wavelengths in the thermal spectrum of a black body radiation, which is shown in below diagram



The results are as follows:

→ At a given temperature, the energy is not uniformly distributed. The intensity of radiation increases with wavelength and reaches a maximum value at a particular wavelength and decreases further as wavelength increases.

→ The wavelength corresponding to maximum intensity is represented by the peak of the curve, and is known as 'λmax'

→ For all wavelength, intensity increases with temperat-ure.

$$T_3 > T_2 > T_1$$

→ As temperature of body increases, the 'λ_{max}' shifts towards shorter wavelength region.

→ The area under the curve will represent the total intensity of radiation or energy density at a particular temperature, i.e.,

$$\boxed{\text{Area under } (E_\lambda \text{ Vs } \lambda) \text{ curve gives } \int E_\lambda \, d\lambda}$$

Ques: State Stefan's law of radiation.

Ans: Stefan's law gives the total energy radiated by a black body. According to Stefan's law, the total amount of radiant energy by a black body per unit area per unit time due to all wavelengths is directly proportional to the fourth power of absolute temperature.

$$E \propto T^4$$

$$\boxed{E = \sigma T^4}$$

where 'σ' is Stefan's constant having value equal to $5.67 \times 10^{-8}\ W/m^2K^4$.

Ques:- What is Wien's law. [2016-17, 2017-18] V.Imp

or

State Wien's displacement law. [2020-21]

or

Explain Wien's law of thermal radiation.

Ans: Wien's proposed two laws of thermal radiation.

(1) Wien's first law or Wien's displacement law of radiation:

Wien's observed that there is a wavelength at which radiation has maximum intensity at a given temperature. The value of λ_{max} is inversely proportional to the absolute temperature of the body.

$$\boxed{\lambda_{max} \propto \frac{1}{T}}$$

$$\lambda_{max} T = b \text{ (constant)}$$

$$\boxed{\lambda_{max} T = 0.0029 \text{ m-K}}$$

Here, 'b' is known as Wien's Constant.

(2) Wien's second law of radiation or Wien's radiation formula:

According to this law, the total energy density $E_\lambda d\lambda$, i.e., amount of radiant energy emitted by a black body per unit area per unit time for a given wavelengths range 'λ and $\lambda + d\lambda$', at a given temperature is expressed as

$$\boxed{E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left(\frac{d\lambda}{e^{hc/\lambda kT}}\right)}$$

where $h = 6.63 \times 10^{-34}$ J-sec → planck's constant
$c = 3 \times 10^8$ m/sec
$k = 1.38 \times 10^{-23}$ J/K → Boltzmann's constant

This law is applicable for shorter wavelength region :- Limitation of Wien's displacement law.

Ques: State Rayleigh - Jeans law. [2020-21] Imp

Ans: Lord Rayleigh used the classical theories of electromagnetism and thermodynamics to show that the blackbody spectral energy distribution is given by:

$$\boxed{E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda}$$

; where k is Boltzmann's constant

Limitation :- It approaches the data at longer wavelengths, but it deviates at shorter wavelengths.

⇒ Planck's Radiation law:

Ques: Write the assumptions of Planck's hypothesis. Imp

Ans: **Basic assumptions of Planck's hypothesis:** [2018-19]

Planck's treated the atoms of the wall of black body as oscillators and each oscillator has characteristic frequency of oscillation.

The assumptions about atomic oscillators are:

(1). The energy of an oscillator can have only certain discrete values 'E_n' or the energy is quantized.

$$E_n = nh\nu$$

; where 'n' is positive integer called quantum number, 'ν' is frequency of oscillation.

(2) The oscillators emit or absorb energy only in the form of packets of energy ($h\nu$) not continuously, when making a transition from one quantum state to other. The entire energy difference between the initial and final states in the transition is emitted or absorbed as a quantum of radiation.

$$E_2 - E_1 = (n_2 - n_1)h\nu$$

$$\boxed{\Delta E = \Delta n \, h\nu}$$

Question: Write plank's law of radiation, how does it explain Wien's displacement and Rayleigh Jean's law

Ans: Plank's radiation law in terms of frequency is given by

$$\boxed{u_\nu d\nu = \frac{8\pi h\nu^3}{c^3} \frac{d\nu}{(e^{h\nu/kT} - 1)}} \quad \text{— (1)}$$

In terms of wavelength

$$\nu = \frac{c}{\lambda} \quad \& \quad d\nu = -\frac{c}{\lambda^2} d\lambda$$

$$\boxed{u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{(e^{\frac{hc}{\lambda kT}} - 1)}} \quad \text{— (2)}$$

For short wavelength ie λ is small $e^{\frac{hc}{\lambda kT}} \gg 1$

so 1 can be neglected in denominator of above eqn

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-\frac{hc}{\lambda kT}} d\lambda$$

This is Wien's law which agree with Experimental curve at short wavelength region

Rayleigh Jeans law from plank's formula →

For long wavelength ie λ is large $e^{\frac{hc}{\lambda kT}} \simeq 1 + \frac{hc}{\lambda kT}$

eqn (2) becomes $u_\lambda d\lambda = \frac{8\pi hc \, d\lambda}{\lambda^5 \left(1 + \frac{hc}{\lambda kT} - 1\right)}$

$$\boxed{u_\lambda d\lambda = \frac{8\pi kT \, d\lambda}{\lambda^4}}$$

This is Rayleigh - Jeans law, which agree with experimental curve at longer wavelength region.

⇒ Compton effect:

V. Imp

Ques: What is Compton effect? [2016-17]

Or

What is Compton effect? Derive necessary expression for Compton shift. [2018-19, 2020-21]

Or

What is Compton effect? Derive suitable expression for Compton shift $\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$. [2018-19]

Or

What is Compton effect? How does it support the photon nature of light? [2019-20]

Or

Explain the modified and unmodified radiation in Compton Scattering. [2016-17]

Ans: Compton effect: In 1921, Professor A.H. Compton discovered that when a monochromatic beam of high frequency radiation (or X-ray) is scattered by a substance (or electron), the scattered radiation contain the radiation not only of the same wavelength or frequency as that of primary rays but also the radiation of greater wavelength or lower frequency.

The radiations of unchanged wavelength in the scattered light are called unmodified radiation while the radiation of greater wavelength (changed wavelength) are called modified radiation. Hence, the phenomenon giving rise to the modification radiation is called Compton effect.

Quantum Explanation : The explanation was given by Compton which was based on Quantum theory of light. According to quantum theory when photon of energy $h\nu$ strikes with the substance, some of the energy of photon is transferred to the electrons, therefore the energy (or frequen-cy) of photon reduces and wavelength increases.

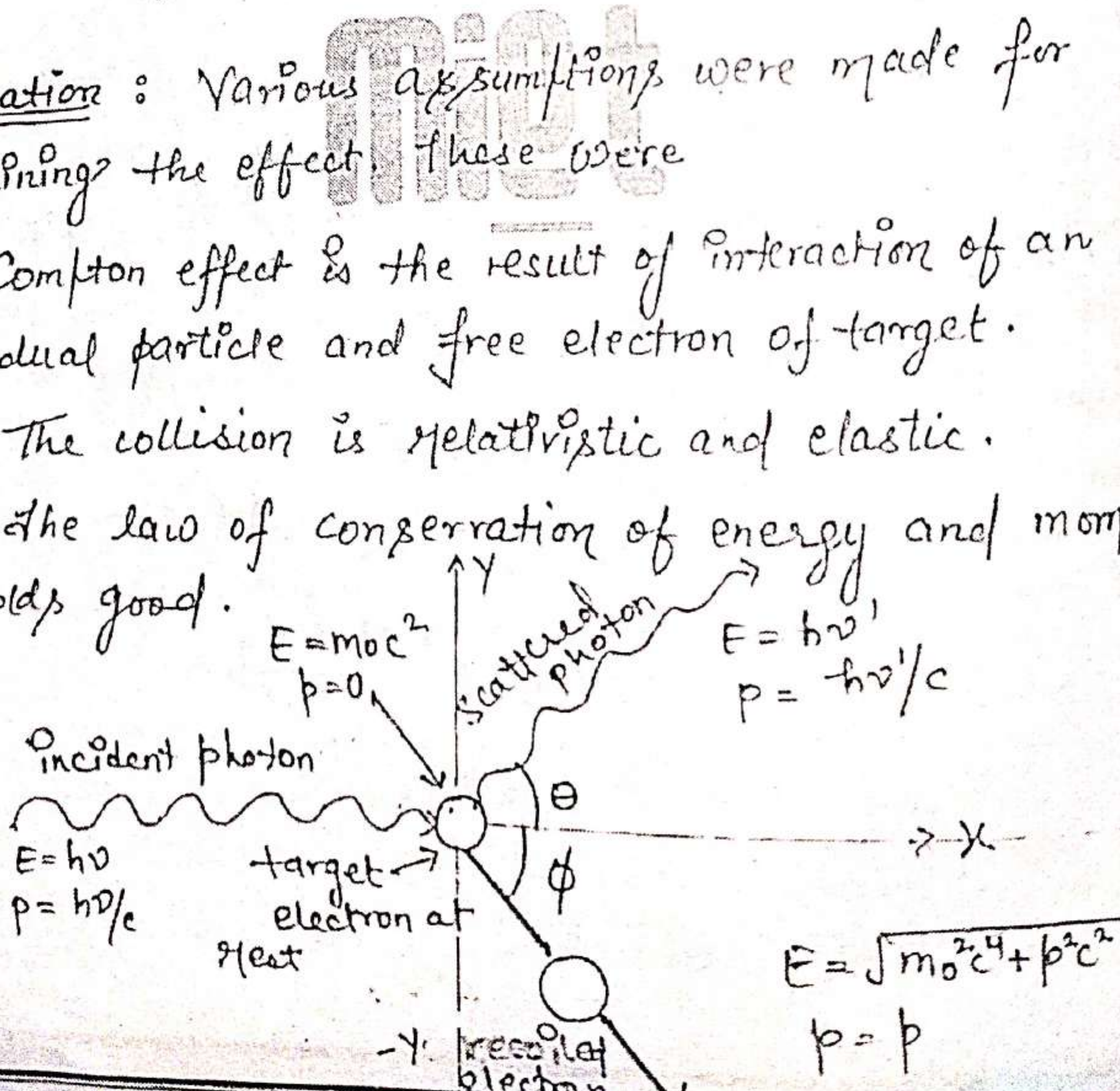
In this way, Compton effect supports photon nature of light.

<u>Derivation</u> : Various assumptions were made for explaining the effect. These were

(i) Compton effect is the result of interaction of an individual particle and free electron of target.

(ii) The collision is relativistic and elastic.

(iii) The law of conservation of energy and momentum holds good.



The energy of the system before collision = $h\nu + m_0c^2$

The energy of the system after collision = $h\nu' + mc^2$

According to the principle of conservation of momentum,

$$h\nu + m_0c^2 = h\nu' + mc^2$$

or $$mc^2 = (h\nu - h\nu') + m_0c^2 \quad \text{------ (1)}$$

According to the principle of conservation of linear momentum along and perpendicular to the direction of incident photon (i.e, along x and y axis), we have

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + mv\cos\phi \quad \text{(along x axis)}$$

or

$$mvc\cos\phi = h\nu - h\nu'\cos\theta \quad \text{------ (2)}$$

Along y-axis,

$$0 + 0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi$$

or $$mvc\sin\phi = h\nu'\sin\theta \quad \text{----- (3)}$$

Squaring (2) and (3) and then adding, we get

$$m^2v^2c^2 = (h\nu - h\nu'\cos\theta)^2 + (h\nu'\sin\theta)^2$$

$$m^2v^2c^2 = (h\nu)^2 + (h\nu'\cos\theta)^2 - 2(h\nu)(h\nu')\cos\theta + (h\nu'\sin\theta)^2$$

$$m^2v^2c^2 = (h\nu)^2 + (h\nu')^2[(\cos\theta)^2 + (\sin\theta)^2] - 2(h\nu)(h\nu')\cos\theta$$

$$m^2v^2c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\theta \quad \text{---- (4)}$$

Squaring eq^n (1), we get

$$m^2c^4 = m_0^2c^4 + (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu') + 2m_0c^2(h\nu - h\nu') \quad ----(5)$$

Subtracting (4) from (5), we get

$$m^2c^4 - m^2v^2c^2 = m_0^2c^4 + 2(h\nu)(h\nu')(\cos\theta - 1) + 2m_0c^2(h\nu - h\nu') \quad ----(6)$$

According to the theory of relativity,

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} \quad \text{or} \quad m^2 = \frac{m_0^2}{(1-v^2/c^2)} \quad \text{or} \quad m^2(1-v^2/c^2) = m_0^2$$

Thus, $m^2c^2 - m^2v^2 = m_0^2c^2 \quad ------(7)$

Multiplying both sides by c^2, we get

$$m^2c^4 - m^2v^2c^2 = m_0^2c^4 \quad --------(8)$$

Using eq^n (8) in (6), we get

$$m_0^2c^4 = m_0^2c^4 + 2(h\nu)(h\nu')(\cos\theta - 1) + 2m_0c^2(h\nu - h\nu')$$

$$0 = 2(h\nu)(h\nu')(\cos\theta - 1) + 2m_0c^2(h\nu - h\nu')$$

$$2(h\nu)(h\nu')(1-\cos\theta) = 2m_0c^2(h\nu - h\nu')$$

$$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta$$

or $\boxed{\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta)} \quad ----(9)$

→ This is compton shift in terms of frequency.

Now, $\because$ $\nu' = \frac{c}{\lambda'}$ and $\nu = \frac{c}{\lambda}$

$\therefore$ we have

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

Compton Shift, $\boxed{\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)}$ in terms of wavelength. ----(10)

Case 1: When $\theta = 0$; $\cos\theta = 1$

$$\therefore \quad \Delta\lambda = \lambda' - \lambda = 0$$

$\Rightarrow \lambda' = \lambda$, the scattered wavelength is same as the incident wavelength in the direction of incidence.

Case 2: When $\underline{\theta = 90^\circ}$, $\underline{\cos\theta = 0}$

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}$$

$$\Delta\lambda = \frac{h}{m_0 c} = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}$$

$$\boxed{\Delta\lambda = \frac{h}{m_0 c} = 0.0243\,A^\circ = \lambda_c}$$

Compton wavelength.

Case 3: When $\theta = 180^\circ$;

$$\Delta\lambda = \lambda' - \lambda = \frac{2h}{m_0 c} = 0.0486\,\mathring{A}$$

Ques: What are the conclusions can be drawn from compton scattering experiment?

Ans: Compton shift is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

It is concluded from above equation that

(1) Wavelength of scattered photon λ' is greater than the wavelength of incident photon.

(2) $\Delta\lambda$ only depends on the scattering angle 'θ'.

(3) $\Delta\lambda$ have the same value for all substance containing free electron.

Ques: Obtain an expression for finding direction and kinetic energy of recoiled electron.

Ans: Direction of recoiled electron:

From compton effect, we have eqn

$$mvc\cos\phi = h\nu - h\nu'\cos\theta \quad \text{-----(1)}$$

and

$$mvc\sin\phi = h\nu'\sin\theta \quad \text{-----(2)}$$

Divide eqn (2) by (3), direction of recoil election is given by

$$\tan\phi = \frac{h\nu'\sin\theta}{h\nu - h\nu'\cos\theta}$$

$$\boxed{\tan\phi = \frac{\nu'\sin\theta}{\nu-\nu'\cos\theta} \quad \text{or} \quad \tan\phi = \frac{\lambda\sin\theta}{\lambda'-\lambda\cos\theta}}$$

Kinetic energy of recoil electron:

$$E = h\nu - h\nu'$$

$$E = \frac{hc}{\lambda} - \frac{hc}{\lambda'}$$

or $$\boxed{E = hc\left(\frac{\lambda' - \lambda}{\lambda'\lambda}\right)}$$

Ques: Why compton effect is not observed in visible spectrum? Imp

Ans: The maximum change in wavelength $\Delta\lambda_{max}$ is 0.0465Å or roughly 0.05Å. This is very small, therefore cannot be observed for wavelength longer than few angstrom units.

For visible radiation, the incident radiation is about 5000Å, $\Delta\lambda_{max}$ is 0.05Å. Therefore, the percentage of incident radiation is about 0.001% which is not detectable.

Wave - particle Duality -

Question: What do you mean by wave- particle duality

Ans : Wave particle duality means electromagnetic radiation has dual character ie in certain situation it exhibits wave properties and in other it acts like a particle. The particle and wave properties of radiation can never be observed simultaneously.

Example of wave nature →

Interference, Diffraction & Polarization

Example of particle nature →

photoelectric effect, Compton effect, black body radiation etc

⇒ Matter Waves:

Ques: What is the concept of de-Broglie matter waves. [2017-18]

Ans: de-Broglie hypothesis of Matter waves:

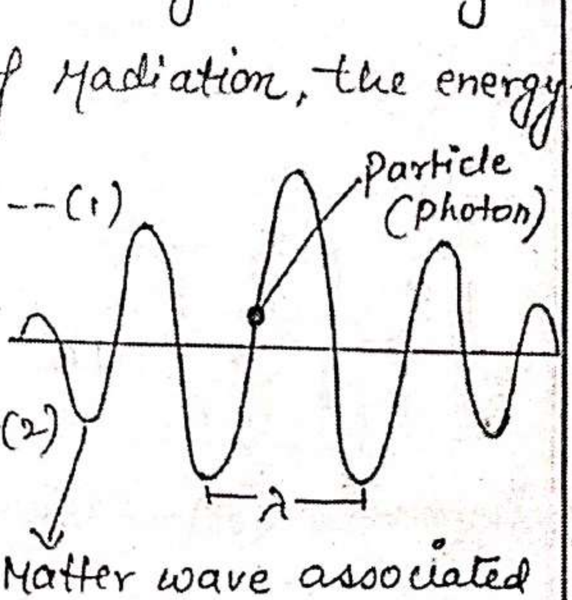
The wave associated with a moving material particle is called matter waves or de-Broglie waves. According to de-Broglie concept (1924), a moving particle always have a wave associated with it. If a particle of mass 'm' has momentum 'p' and 'λ' is the wavelength of wave associated with it, then according to de-Broglie hypothesis

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}} \rightarrow \text{for materialistic particle}$$

Ques: Obtain an expression for de-Broglie wavelength of photon. [2018-19]

Ans: According to Planck's theory of radiation, the energy of photon is given by

$$E = h\nu \quad \text{----(1)}$$



Matter wave associated with moving particle (photon)

According to Einstein energy-mass relation,

$$E = mc^2 \quad \text{---(2)}$$

from eq^n (1) and (2), we get

$$h\nu = mc^2$$

$$\frac{hc}{\lambda} = mc^2 \quad [\because c = \nu\lambda]$$

$$\frac{h}{\lambda} = mc$$

$$\boxed{\lambda = \frac{h}{mc}}$$ — Required de-Broglie wavelength expression for photon.

Ques: Obtain a relation between
(1) de-Broglie wavelength and kinetic energy,
(2) de-Broglie wavelength and temperature.

Ans: (1) de-Broglie wavelength is given by.

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

If 'E' is kinetic energy of material particle, then

$$E_K = \frac{1}{2}m.v^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$$

$$p = \sqrt{2mE_K}$$

$$\therefore \boxed{\lambda = \frac{h}{\sqrt{2mE_K}}}$$

(2) When a material particle is in thermal equilibrium at temperature 'T', then

$$E = \frac{3}{2}kT$$ (k is Boltzmann constant)

∴ de-Broglie wavelength at temp. 'T' is given by

$$\lambda = \frac{h}{\sqrt{2mE_K}} \Rightarrow \boxed{\lambda = \frac{h}{\sqrt{3mkT}}}$$

Ques: Obtain de-Broglie wavelength associated with electrons.

Ans: Let us consider the case of an electron of rest mass 'm_0' and charge 'e' which is accelerated by a potential 'V' volt from rest to velocity v, then $\boxed{E_K = eV}$

$$\frac{1}{2}mv^2 = eV \quad \text{or} \quad v = \sqrt{2eV/m_0}$$

$$\therefore \quad \lambda = \frac{h}{p} = \frac{h}{m_0 v} = \frac{h}{\sqrt{2eV/m_0}}$$

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times1.6\times10^{-19}\times V\times 1.91\times10^{-31}}}$$

$$\Rightarrow \boxed{\lambda = \frac{12.28}{\sqrt{V}}\,\text{Å}}$$

<u>Note</u>: For any charged particle carrying charge 'q', accelerated by potential difference 'V' volts,

$$E_K = qV$$

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mE_K}} \Rightarrow \boxed{\lambda = \frac{h}{\sqrt{2mqV}}}$$

Ques: Why are matter waves associated with a particle generated only when it is in motion? [2020-21] Imp

Ans: de-Broglie wavelength is given by.

$$\lambda = \frac{h}{mv}$$

When $v = 0$, then $\lambda = \infty$, i.e, wave becomes indeterminate or we cannot define the wave nature of particle and if $v = \infty$, then $\lambda = 0$. This shows that waves are generated by the motion of particles.

Phase Velocity and Group Velocity

→ When a monochromatic wave (single frequency & wavelength) travels through a medium, its velocity of advancement in the medium is called as phase or wave velocity

· Consider a wave whose displacement y is expressed as,

$$y = a\sin(\omega t - Kx)$$

∴ a = amplitude
∵ ω = angular frequency
∴ K = wave vector

Here, $(\omega t - Kx)$ is phase of wave motion.

For planes of constant phase, $\omega t - Kx = \text{constant}$

— differentiating wrt. t —

$$\omega - K\frac{dx}{dt} = 0$$

$$\Rightarrow \boxed{\frac{dx}{dt} = V_p = \frac{\omega}{K}}$$ → phase velocity/ wave velocity

Thus, the wave velocity is the velocity with which the planes of constant phase advance through the medium. So, it is also called phase velocity.

⇒ A particle in motion has two velocities: particle velocity (v) and its associated matter wave velocity (v_p).

We know that, $E = h\nu = mc^2 \Rightarrow \nu = \frac{mc^2}{h}$

The wave velocity is given by, $V_p = \nu\lambda$ $\quad\left(\because \lambda = \frac{h}{mv}\right)$

$$V_p = \left(\frac{mc^2}{h}\right)\left(\frac{h}{mv}\right)$$

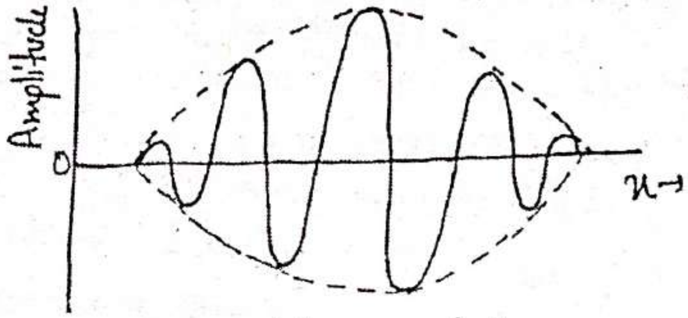
$$\boxed{V_p = \frac{c^2}{v}}$$

∴ c = velocity of light

As $V_p > c$ which is in direct contradiction with Special Theory of Relativity. Thus V_p can not be greater than velocity of light.

Group Velocity

Wave packet A wave packet comprises a group of waves slightly differing in velocity and wavelength such that they interfere constructively over a small region of space where the particle can be located and outside this space they interfere destructively so that the amplitude reduces to zero.



Wave packet

The velocity with which the wave packet obtained by superposition of wave travelling in group is called group velocity (v_g).

$$\boxed{V_g = \frac{d\omega}{dK}}$$

Relation between group velocity & wave velocity (for dispersive medium)

Wave velocity, $V_p = \frac{\omega}{K} \Rightarrow \omega = V_p K$

$$\Rightarrow d\omega = V_p\, dK + K\, dV_p$$

$$\Rightarrow d\omega = dK\left(V_p + K\frac{dV_p}{dK}\right)$$

$$\Rightarrow \frac{d\omega}{dK} = V_p + K\frac{dV_p}{dK}$$

$\because \frac{d\omega}{dK}$ = group velocity

$$\Rightarrow \boxed{V_g = V_p + K\frac{dV_p}{dK}} \quad -①$$

Also, $K = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{K}$

$$\left.\begin{aligned} &\Rightarrow \frac{d\lambda}{dK} = -\frac{2\pi}{K^2} \end{aligned}\right] \quad -②$$

From ① ; $v_g = v_p + k\dfrac{dv_p}{dk}$

$$v_g = v_p + k\frac{dv_p}{d\lambda}\left(\frac{d\lambda}{dk}\right) \qquad \therefore \text{using } ②$$

$$v_g = v_p + \left(\cancel{k}\frac{2\pi}{\lambda}\right)\cdot\frac{dv_p}{d\lambda}\left(-\frac{2\pi}{k^2}\right)$$

$$v_g = v_p - \frac{4\pi^2}{\lambda}\cdot\frac{1}{k^2}\frac{dv_p}{d\lambda} \qquad \therefore \frac{1}{k^2} = \frac{\lambda^2}{4\pi^2}$$

$$\boxed{v_g = v_p - \lambda\frac{dv_p}{d\lambda}} \quad - ③$$

For non-dispersive medium, $\dfrac{dv_p}{d\lambda} = 0$ Thus, $\boxed{v_g = v_p}$

Equation ③ represent relationship between phase and group velocities in which the phase velocity is frequency ~~in~~dependent.

Phase velocity (or wave velocity) of de-Broglie waves

The propagation vector of a wave, $k = \dfrac{2\pi}{\lambda}$ $\left(\therefore \text{de Broglie wavelength } \lambda = \dfrac{h}{mv}\right)$

$$k = \frac{2\pi}{h}(mv) \quad - ①$$

If E is the energy of particle corresponding to frequency ν

Then, $E = h\nu \Rightarrow \cancel{\nu = \frac{h}{E}}\ \nu = \dfrac{E}{h}$

$\therefore$ Angular frequency, $\omega = 2\pi\nu = 2\pi\left(\dfrac{E}{h}\right)$ $\quad (\therefore E = mc^2)$

$$\omega = \frac{2\pi mc^2}{h} \quad - ②$$

Now, de-Broglie phase velocity (wave velocity),

$$v_p = \frac{\omega}{k} \quad - ③$$

Putting the values of K & ω from (1) & (2) in (3) we have,

$$V_p = \frac{\omega}{K} = \left(\frac{2\pi mc^2}{h}\right) \times \left(\frac{h}{2\pi mv}\right)$$

$$V_p = \frac{c^2}{v}$$

∴ The particle is constitutes all sort of wave group. So, $v = v_g$

$$\Rightarrow \quad V_p = \frac{c^2}{v_g}$$

$$\Rightarrow \quad \boxed{V_p V_g = c^2}$$

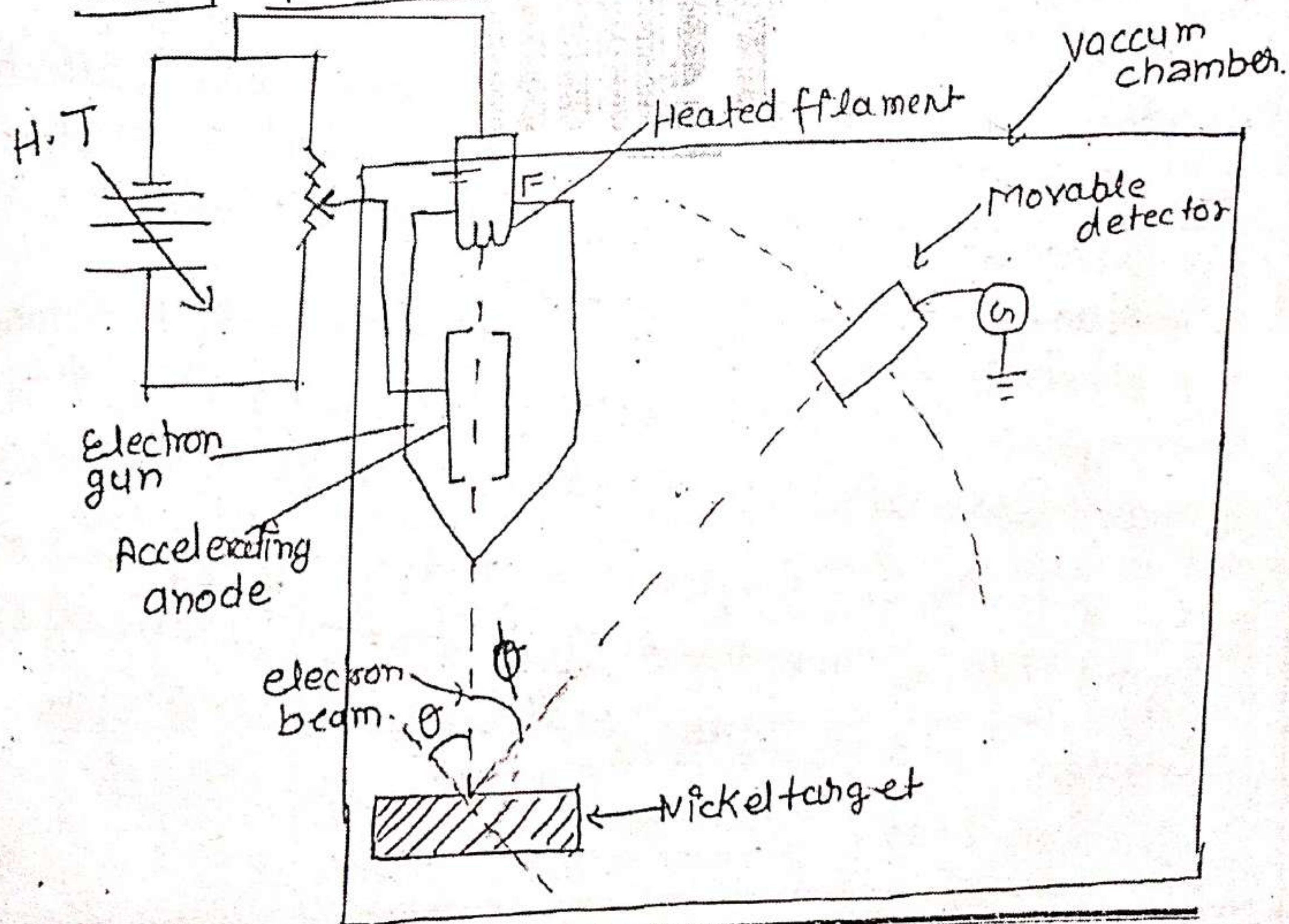
Thus, the product of phase velocity (v_p) and group velocity (v_g) is equal to the square of velocity of light (c^2)

∴ As particle velocity $v = v_g < c$ So, de-Broglie wave velocity $v_p > c$.
So, the de-Broglie wave train associated with the particle would travel much faster than the particle itself and would leave the particle far behind.
This statement is nothing but the collapse of the wave description of the particle.

* **Davisson and Germer Experiment:-** It is the first experimental evidence of material particle was predicted in 1927 by Clinton Davisson and Lester Germer. This experiment not only confirmed the existence of waves associated with electron by detecting de-broglie waves but also succeed in measuring the wavelength.

- Davisson and Germer's experiment were originally designed for the study of scattering of electrons by a nickel crystal.

* **Set-Up of Davisson-Germer Experiment**



* **Procedure:-** The electron are produced by thermionic emission from a tungsten filament F mounted in an electron gun. The ejected electrons are accelerated towards anode in an electric field of known potential difference and collimated into a narrow beam. The whole arrangement used to emit electron and to accelerate and to focus is called electron gun.

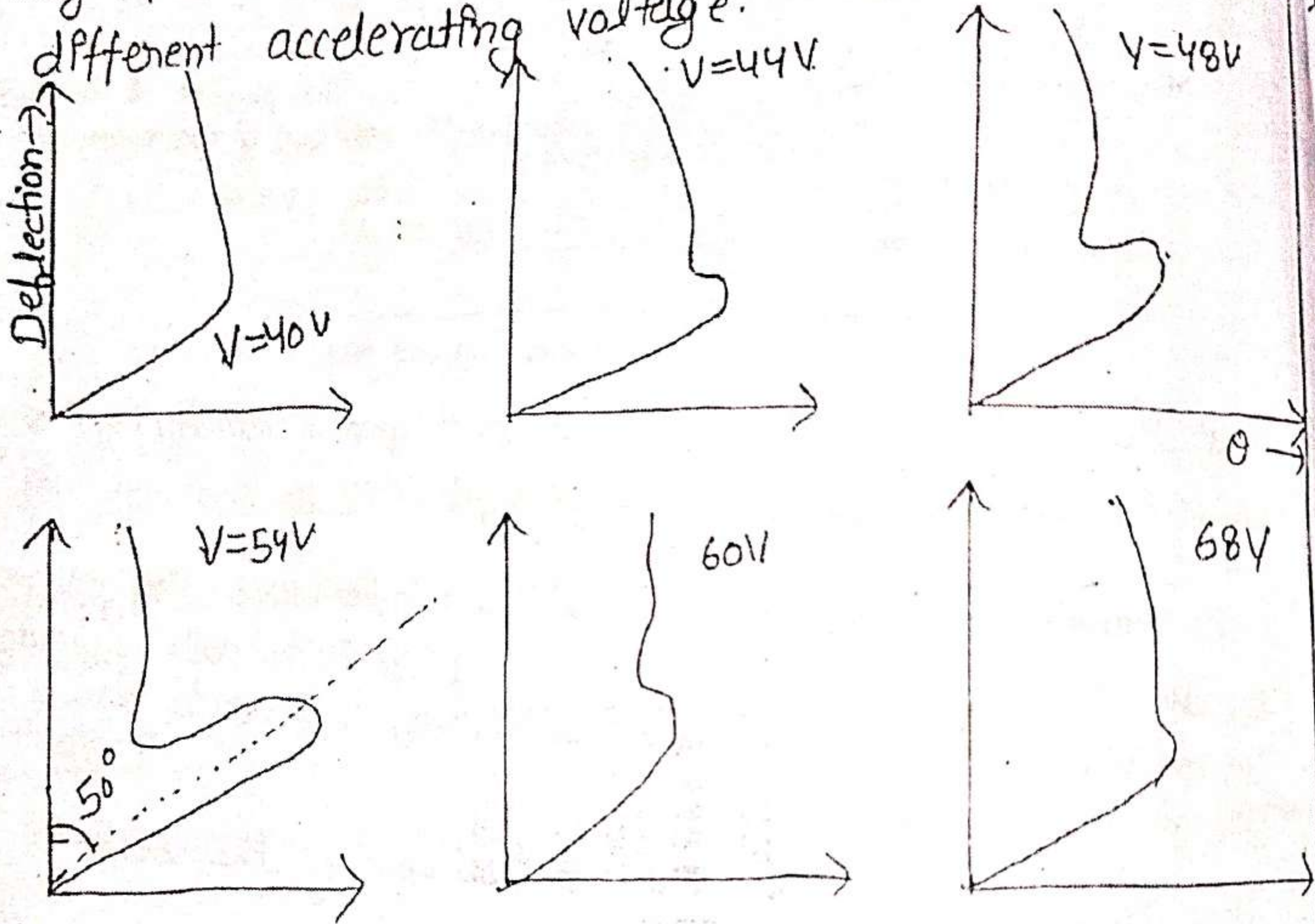
The narrow beam of electron is allowed to fall normally on the surface of a nickel crystal because their atoms are arranged in a regular pattern / lattice. So the surface lattice of the crystal acts as a diffraction grating and electrons are diffracted by crystal in different direction.

The electron scattered from target are collected by detector C which is also connected to a Galvanometer G and can be moved along a circular scale S.

The electron are scattered in all direction by atoms in crystal. The intensity of electrons scattered in a particular direction is formed by using a detector.

On rotating the detector, the intensity of scattered beams can be measured for different value of angle between incident and scattered direction of electron beam.

The various graphs are plotted between Scattering angle ϕ and intensity of scattered beam at different accelerating voltage.



*Salient Features Observed from the Graph.

1.) Intensity of scattered electron depends upon angle of scattering ϕ.
2.) A Kink begins to appear in curve at 44 Volts.
3.) This Kink moves upward as the voltage increases and become more prominent for 54 volts at $\phi = 50^\circ$.
4.) The size of Kink starts decreasing with further increase in accelerating voltage and drops almost to zero at 68 Volts.
5.) The Kink at 54 Volt offer evidence for existence of electron waves.

The crystal surface acts like a diffraction grating with spacing d.

The principal maxima for such a grating must satisfy Bragg's equation.

$$2d\sin\theta = n\lambda$$

where $d \rightarrow$ interplanar spacing

$d = 0.91 A^{0}$

$n = 1$ for 1st order.

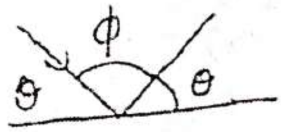
Given, Angle of diffraction $= \phi$

glancing angle $= \theta$.

According to Davisson - Germer experiment from figure

$2\theta + \phi = 180^{0}$

$\theta = 90 - \frac{\phi}{2}$



fig

for $\phi = 50^{0}$, $\theta = 90 - \frac{50}{2} = 90 - 25$

$\boxed{\theta = 65^{0}}$ $2d\sin\theta = \lambda$

$2 \times 0.91 \times 10^{-10} \sin 65^{0} = \lambda$

$\lambda = 1.65 \times 10^{-10} m$ i.e $\boxed{\lambda = 1.65 A^{0}}$

Now, For electron kinetic energy of 54eV

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 54 \times 1.6 \times 10^{-19}}}$$

$\lambda = 1.67 \times 10^{-10} m$

$\boxed{\lambda = 1.67 A^{0}}$

which agrees well with observed wavelength of $1.65 A^{0}$..

Thus, Davisson - Germer experiment directly verifies de-broglie hypothesis of wave nature of moving bodies

Ques: What is the difference between electromagnetic wave and matter wave? [2019-20] Imp

Ans:

	Electromagnetic Wave	Matter Wave
1.	Electromagentic wave are associated with accelerated charge particle.	1. Matter waves are associated with moving particle and does not depend on the charge.
2.	EM waves are the type of wave that travels through space, carrying electromagnetic radiant energy.	2. Matter wave are the waves that consist of particles.
3.	EM waves have electric and magnetic fields associated with them.	3. Matter waves do not have any associated electric and magnetic field.
4.	EM waves consists of photons (which have no mass or volume.	4. Matter waves contain particles (which have mass and volume).

Ques: Interpret Bohr's quantization rule on the basis of de-Broglie concept of matter waves. [2019-20]

Ans: Since, the electron does not radiate energy while moving in its orbit, the wave associated with it must be stationary wave in which there is no loss of energy.

Thus, the election forms the standing wave only when circumference of orbit is integral multiple of wavelength, i.e.,

$$\boxed{2\pi r_n = n\lambda} \quad \text{——— (1)}$$

$\therefore \lambda = h/mv$

$\therefore$ from eq^n (1), $2\pi r = \frac{nh}{mv}$ ——— (2)

$$r = \frac{nh}{2\pi mv} \quad \text{——— (3)}$$

Now, Angular momentum is given by.

$$\boxed{L = mvr \text{ or } L = \frac{nh}{2\pi}} \quad \left[\text{using eq (2)}\right]$$

This is the Bohr's quantization condition. According to which, an electron can revolve only in certain discrete orbits, for which total angular momentum of the revolving electron is an integral multiple of $h/2\pi$, where 'h' is planck's constant.

Ques: Give physical interpretation of wave function. Also, explain eigenvalues and eigenfunction. [2016-17] V. Imp

or

Discuss the physical significance of wave function. [2018-19]

Ans: The quantity whose variation builts up matter waves is called wave function. The satisfactory interpretation of the wave function 'ψ' was given by Born in 1926.

He postulated that the square of the magnitude of the wave function, $|\psi|^2$ or $\psi^*\psi$ (if ψ is complex) associated with a moving particle at a particular point (x, y, z) in space at the time 't' represents the probability of finding the particle at the point. $|\psi|^2$ is called probability density and 'ψ' is 'probability' amplitude.

According to this interpretation, the probability of finding the particle within an element of volume $d\tau$ is $|\psi|^2 d\tau$. Since the particle is certainly somewhere in space, the total probability of finding the particle in space is unity, i.e.,

$$\boxed{\iiint |\psi|^2 dx\,dy\,dz = 1}$$

→ This is normalisation condition and 'ψ' is said to be normalized.

The value of energy for which Schrodinger equation (time independent) can be solved are called 'eigen values' and the corresponding (acceptable) wave function are called "eigenfunction".

Ques: Mention the characteristic of wave function.

Ans. (1) It must be normalized.
(2) It must be finite everywhere.
(3) It must be single - valued.
(4) It must be continuous and its first derivative should also be continuous.

Ques: Show that $\psi(x,y,z,t) = \psi(x,y,z)e^{-i\omega t}$ is a function of stationary state. [2018-19]

Ans: If 'ψ' is a wavefunction of stationary state, then the value of $|\psi|^2$ is independent of time. So,

$$\psi(x,y,z,t) = \psi(x,y,z)e^{-i\omega t}$$

$$|\psi|^2 = \psi^*\psi = \psi(x,y,z)e^{-i\omega t}\,\psi^*(x,y,z)e^{i\omega t}$$

$$|\psi|^2 = \psi(x,y,z)\,\psi^*(x,y,z)$$

$$|\psi(x,y,z,t)|^2 = |\psi(x,y,z)|^2 \quad \text{——— (1)}$$

Above equation shows that $|\psi|^2$ is not a function of time, so $\therefore$ $\psi(x,y,z,t) = \psi(x,y,z)e^{-i\omega t}$ is a function of stationary state.

⇒ Schrodinger wave equation:

Ques: Obtain time dependent and time independent wave equation. [2018-19] V. Imp

Or

Derive time independent Schrodinger wave equation. [2016-17, 2020-21]

Ans: **Schrodinger time dependent wave equation:**

The differential equation of a wave motion of a particle in one-dimensional can be written as:

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \quad \text{--------}(1)$$

The general solution of eq^n (1) is given by

$$\psi(x,t) = Ae^{-i\omega[t-(x/v)]} \quad \text{--------}(2)$$

We know that $\omega = 2\pi\nu$ and $v = \nu\lambda$. So, eq (2) can be written as,

$$\psi(x,t) = Ae^{-2\pi i[\nu t-(x/\lambda)]} \quad \text{--------}(3)$$

Put $\nu = E/h$ and $\lambda = h/p$ in eq^n (3), we get

$$\psi(x,t) = Ae^{-(2\pi i/h)(Et-px)} \quad \text{------}(4)$$

Now, differentating eq^n (4) twice with respect to x, we get

$$\frac{\partial^2 \psi}{\partial x^2} = Ae^{-\left(\frac{2\pi i}{h}\right)(Et-px)}\left(\frac{2\pi i p}{h}\right)^2$$

$$= -\frac{4\pi^2 p^2}{h^2}\psi$$

$$p^2 = -\frac{1}{\psi}\frac{h^2}{4\pi^2}\frac{\partial^2\psi}{\partial x^2} \quad \text{--------(5)}$$

Now, differentiate eq^n (4) with respect to t, we get

$$\frac{\partial\psi}{\partial t} = A e^{-(2\pi i/h)(Et - px)}\left(-\frac{2\pi i E}{h}\right)$$

$$= -\frac{2\pi i E}{h}\psi$$

$$\text{or} \quad E = -\frac{1}{\psi}\frac{h}{2\pi i}\frac{\partial\psi}{\partial t} = \frac{1}{\psi}\frac{ih}{2\pi}\frac{\partial\psi}{\partial t} \quad \text{-----(6)}$$

If E and V be the total and potential energies of the particle respectively, then its Kinetic energy $\frac{1}{2}mv^2$ is given by

$$E = \frac{1}{2}mv^2 + V = \frac{1}{2}\frac{m \cdot mv^2}{m} + V$$

$$\text{or} \quad E = \frac{p^2}{2m} + V \quad \text{--------(7)}$$

Putting the values from eq^n (5) and (6) in eq^n(7), we get

$$\frac{1}{\psi}\frac{ih}{2\pi}\frac{\partial\psi}{\partial t} = \frac{h^2}{8\pi^2 m}\frac{1}{\psi}\frac{\partial^2\psi}{\partial x^2} + V$$

$$\text{or} \quad -\frac{h^2}{8\pi^2 m}\frac{\partial^2\psi}{\partial x^2} + V\psi = \frac{ih}{2\pi}\frac{\partial\psi}{\partial t}$$

Substituting $\hbar = h/2\pi$, we get

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}} \quad \text{------(8)}$$

$\hookrightarrow$ Required Schrodinger time dependent equation in one dimension.

In 3-D, the above equation can be written as:

$$\boxed{\left(-\frac{\hbar^2}{2m}\nabla^2+V\right)\psi = i\hbar\frac{\partial\psi}{\partial t}}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$

or $\boxed{H\psi = E\psi}$

where $H = -\frac{\hbar^2}{2m}\nabla^2+V \longrightarrow$ Hamiltonian Operator

$E = \frac{ih}{2\pi}\frac{\partial}{\partial t} \longrightarrow$ Energy Operator

Schrodinger's time independent equation:

This eq^n can be obtained with the help of time dependent equation. The differential equation of a wave motion of a particle in one-dimension can be written as

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{------}(1)$$

The general solution of eq^n(1) is given by

$$\psi(x,t) = A\,e^{-i\omega[t-(x/v)]} \quad \text{------}(2)$$

We know that $\omega = 2\pi\nu$ and $v = \nu\lambda$. So, eq (2) can be written as

$$\psi(x,t) = A\,e^{-2\pi i[\nu t-(x/\lambda)]} \quad \text{------}(3)$$

Put $\nu = E/h$ and $\lambda = h/p$ in eq^n (3), we got

$$\psi'(x,t) = A e^{-(2\pi i/h)(Et - px)} \quad \text{------(4)}$$

The wave function can be separated into time dependent and space dependent parts.

$$\psi(x,t) = A e^{(2\pi i p x/h)} e^{-(2\pi i Et/h)}$$

If $\psi(x) = \psi_0 = A e^{(2\pi i p x/h)}$, then

$$\psi(x,t) = \psi_0 e^{-(2\pi i Et/h)} \quad \text{------(5)}$$

Now, differentiating eq^n (5) twice with respect to x, we get

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 \psi_0}{\partial x^2} e^{-\left(\frac{2\pi i Et}{h}\right)} \quad \text{------(6)}$$

Differentiating eq^n (5) with respect to 't', we get

$$\frac{\partial \psi}{\partial t} = \psi_0 e^{-(2\pi i Et/h)} \left(-\frac{2\pi i E}{h}\right) \quad \text{----(7)}$$

Using Schrodinger time dependent equation in one-dimension, i.e.,

$$-\frac{h^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = \frac{ih}{2\pi}\frac{\partial \psi}{\partial t}$$

Substitute value of ψ, $\frac{\partial^2 \psi}{\partial x^2}$ and $\frac{\partial \psi}{\partial t}$ from equation (5), (6) and (7) in above equation,

$$-\frac{h^2}{2m}\frac{\partial^2 \psi_0}{\partial x^2} e^{-(2\pi i Et/h)} + V\psi_0 e^{-(2\pi i Et/h)} = \frac{ih\psi_0}{2\pi} e^{-\left(\frac{2\pi i Et}{h}\right)} \cdot \left(-\frac{2\pi i E}{h}\right)$$

or $$-\frac{h^2}{2m}\frac{\partial^2 \psi_0}{\partial x^2} + V\psi_0 = E\psi_0$$

$$\boxed{\frac{\partial^2 \psi_0}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi_0 = 0}$$

Required time independent equation in one dimension.

Or generally,

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$

In 3-D, above equation can be written as -

$$\boxed{\nabla^2 \psi + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$

Note :- For a free particle,

$$\boxed{V = 0}$$

$\therefore$ Schrodinger wave equation for a free particle can be expressed as,

$$\boxed{\nabla^2 \psi + \frac{2m}{\hbar^2} E\psi = 0}$$

⇒ Particle in One-dimensional box : V·Imp

Ques: Find an expression for the energy states of a particle in a one-dimensional box. [2017-18]

or

Write down Schrodinger wave equation for particle in a one dimensional box and solved it to find out the Eigen value and Eigen function. [2019-20]

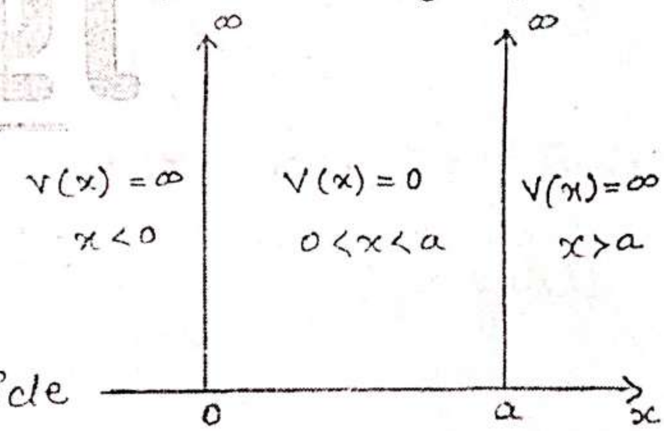
Ans: Particle in one-dimension box :

Let us consider the case of a particle of mass 'm' moving along x-axis between two rigid walls A and B at $x = 0$ and $x = a$. The particle is free to move between the walls.

The potential function is defined is defined as

$V(x) = 0$ for $0 < x < a$

$V(x) = \infty$ for $0 \geq x$ and $x \geq a$



Under this condition, particle is said to move in an infinitely deep potential well or infinite square well.

The Schrodinger equation for the particle within the box (V=0) is,

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E\psi = 0} \quad \text{---(1)}$$

or $\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0$ ---------(2)

where $\boxed{k^2 = 2mE/\hbar^2}$ ------ (3)

The general solution of equation (2) is given as

$\psi = A \sin kx + B \cos kx$ -----(4)

Apply the boundary condition, $\psi = 0$ at $x = 0$ and $x = a$, to eqn (4), $\therefore$ $0 = A \sin k \cdot 0 + B \cos k \cdot 0$

$\Rightarrow \boxed{B = 0}$

Again at $x = a$, $\psi = 0$,

$\therefore A \sin ka = 0$

$\boxed{ka = \pm n\pi}$ ---(5) where $n = 1, 2, 3 \dots$

$n \neq 0$, because for $n = 0$, $k = 0$

Using eqn (3) and eqn (5), we get

$$\boxed{E_n = \frac{n^2 \hbar^2 \pi^2}{2ma^2}}$$ ......(6)

or $\boxed{E_n = \frac{n^2 h^2}{8ma^2}}$ ----(6) $\left[\because \hbar = \frac{h}{2\pi}\right]$

From eqn (6), it is clear that particle cannot have an arbitrary energy, but can have certain discrete energy corresponding to $n = 1, 2, 3, \dots$. Each permitted energy is called Eigen value of the particle and constitute the energy level of the system. The corresponding eigen function is given by $\psi = A \sin kx$

Applying normalisation condition for finding the value of A,

$$\int_0^a |\psi_n|^2 dx = 1$$

$$A^2 \int_0^a \sin^2 \frac{n\pi x}{a} dx = 1$$

$$A^2 \int_0^a \frac{1}{2} \left\{ 1 - \cos\left(\frac{2n\pi x}{a}\right) \right\} dx = 1$$

$$\left[\because \sin^2\theta = \frac{1 - \cos 2\theta}{2} \right]$$

$$\frac{A^2}{2} \left[x - \frac{\sin(2n\pi x/a)}{(2n\pi/a)} \right]_0^a = 1$$

$$\frac{A^2}{2}(a) = 1$$

$$\Rightarrow \boxed{A = \sqrt{\frac{2}{a}}}$$

∴ The normalised wavefunction for n^{th} state is given by

$$\boxed{\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)}$$

— Required expression for eigen wave function

Ques: Show that energy levels are not equally spaced for the particle in one-dimensional box.

Ans: We know that energy for a particle travelling in one-dimensional box is given by

$$\boxed{E_n = \frac{n^2 h^2}{8ma^2}}$$

Putting $n = 1, 2, 3, 4, 5$ ---- so on, we get

$$E_1 = \frac{n^2h^2}{8ma^2} \quad ; \quad E_2 = \frac{4h^2}{8ma^2} = 4E_1 \quad ; \quad E_3 = \frac{9h^2}{8ma^2} = 9E_1$$

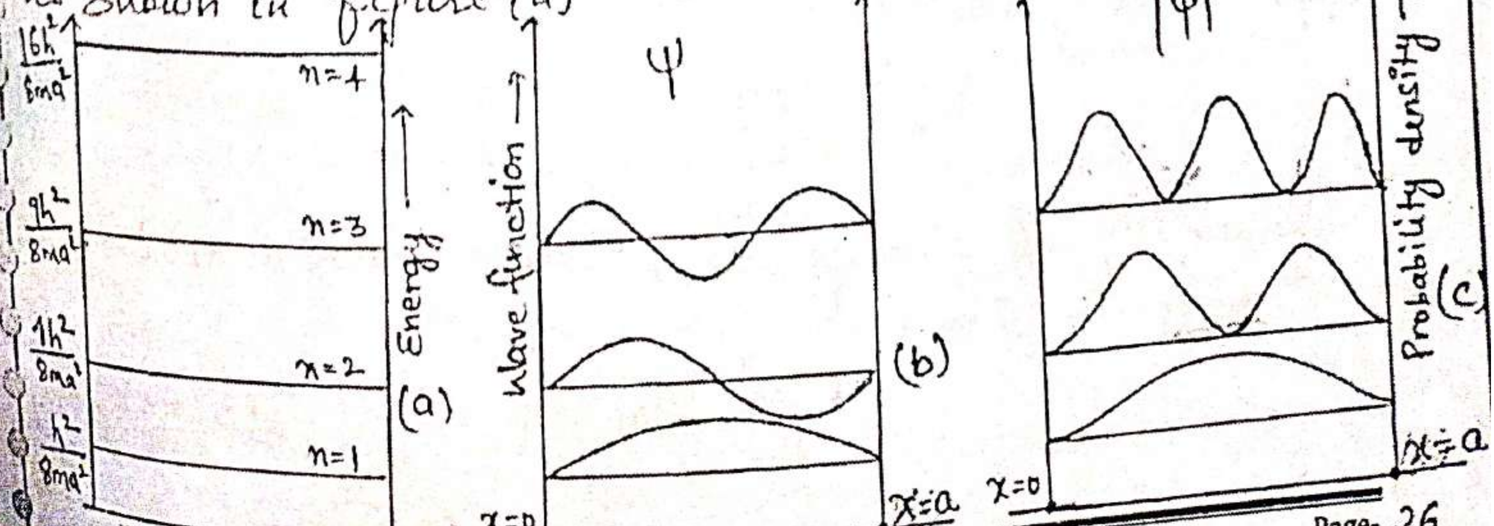
$$E_4 = \frac{16h^2}{8ma^2} = 16E_1 \quad ; \quad E_5 = \frac{25h^2}{8ma^2} = 25E_1$$

Now, the difference between energy in various states

$$\boxed{E_2 - E_1 = 3E_1} \quad ; \quad \boxed{E_3 - E_2 = 5E_1} \quad ; \quad \boxed{E_4 - E_3 = 7E_1}$$

$$\boxed{E_5 - E_4 = 9E_1}$$

From above equation, it is clear that the difference between energy of consecutive energy state is not constant. This difference is increases with increas in value of 'n'. Therefore, it is concluded that the energy levels are not equally spaced as shown in figure (a)



Numericals

Ques: Calculate the energy of oscillator of frequency 4.2×10^{12} Hz at 27°c treating it as (a) classical oscillator (b) Planck's oscillator. [2018-19]

Soln: According to classical theory, the average energy of classical oscillator at temp. 'T' is given by

$$E_{classical} = KT = 1.38 \times 10^{-23} \times 300$$

$$= 4.14 \times 10^{-21} \text{ Joule } \underline{Ans}$$

$$\{T = 27 + 273 = 300K\}$$

According to Quantum Mechanics, average energy of Planck's oscillator is given by

$$E_{Planck} = \frac{h\nu}{e^{h\nu/KT} - 1}$$

$$h\nu = 6.63 \times 10^{-34} \times 4.2 \times 10^{12} = 2.78 \times 10^{-21} \text{ Joule}$$

$$\frac{h\nu}{KT} = \frac{2.78 \times 10^{-21}}{1.38 \times 10^{-23} \times 300} = 0.67$$

$$\therefore e^{h\nu/KT} - 1 = e^{0.67} - 1 = 1.954 - 1 = 0.954$$

$$\therefore E_{Planck} = \frac{h\nu}{e^{h\nu/KT} - 1} = \frac{2.78 \times 10^{-21}}{0.954}$$

$$E_{Planck} = 2.914 \times 10^{-21} \text{ Joule}$$

Ques: What is the wavelength of maximum intensity radiation, radiated from a source having temperature 3000K ? The Wien's constant is 0.3×10^{-2} m-K

Soln: According to Wien's law,

$$\lambda_m T = b$$

$$\lambda_m = b/T$$

$$\therefore \lambda_m = \frac{0.3 \times 10^{-2}}{3000} = 0.1 \times 10^{-5} m$$

$$\boxed{\lambda_m = 10{,}000\,\mathring{A}}$$ Ans

Ques: Calculate the wavelength of an α-particle accelerated through a potential difference of 200 Volts. V. Imp

Soln: The de-Broglie wavelength of an α-particle accelerated through a potential difference 'V' is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

For α-particle,

$$q = 2e = 2 \times 1.6 \times 10^{-19} \text{ Coulomb}$$

$$m = 4 \times \text{mass of proton} = 4 \times 1.67 \times 10^{-27} kg$$

$$\therefore \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 200}}$$

$$\lambda = 7.16 \times 10^{-13} m$$

or $$\boxed{\lambda = 7.16 \times 10^{-3}\,\mathring{A} = 0.00716\,\mathring{A}}$$ Ans.

Ques: Calculate the de-Broglie wavelength of neutron having kinetic energy 1eV. [2019-20] Imp

Solⁿ: The rest energy of neutron is $m_0c^2 = 1.67 \times 10^{-27} \times (3\times10^8)^2$

$= 1.503\times10^{-10}$ J

$= 939.4$ MeV

The kinetic energy of the given neutron is 1eV, i.e; 1.6×10^{-19} J, is very small as compared to its rest mass energy. Therefore, the relativistic consideration may be ignored. So, the de-Broglie wavelength of an neutron of rest mass 'm_0' is given by

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.6\times10^{-19}}}$$

$$\boxed{\lambda = 2.87\times10^{-11}\,m}$$ Ans

Ques: Determine the probability of finding a particle trapped in a box of length L in the region from 0.45L to 0.55L for the ground state. [2017-18]

Solⁿ: The eigen function of a particle trapped in a box of length 'L' is given by

$$\psi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

Probability of finding the particle between x_1 and x_2 for n^{th} state is given by

$$P = \int_{x_1}^{x_2} |\psi_n|^2 dx = \frac{2}{L}\int_{x_1}^{x_2} \sin^2\left(\frac{n\pi x}{L}\right)dx$$

$$P = \frac{2}{L}\int_{x_1}^{x_2} \frac{1}{2}\left\{1 - \cos\left(\frac{2n\pi x}{L}\right)\right\} dx$$

$$P = \frac{1}{L}\left[x - \frac{L}{2\pi n}\sin\left(\frac{2n\pi x}{L}\right)\right]_{x_1}^{x_2}$$

Now, $x_1 = 0.45L$; $x_2 = 0.55L$ and for ground state $n=1$;

$$\therefore P = \frac{1}{L}\left[\left\{0.55L - \frac{L}{2\pi}\sin(1.1\pi)\right\} - \left\{0.45L - \frac{L}{2\pi}\sin(0.9\pi)\right\}\right]$$

$$= \left[\left\{0.55 - \frac{1}{2\pi}\sin(198^\circ)\right\} - \left\{0.45 - \frac{1}{2\pi}\sin(162^\circ)\right\}\right]$$

$$= (0.55 - 0.45) - \frac{1}{2\pi}(\sin 198^\circ - \sin 162^\circ)$$

$$= 0.10 - \frac{1\times 2}{2\pi}(\cos 180^\circ \sin 18^\circ)$$

$$\left[\because \sin A - \sin B = 2\cos\frac{(A+B)}{2}\sin\frac{(A-B)}{2}\right]$$

$$= 0.10 + \frac{1}{3.14}(0.309)$$

$$= 0.10 + 0.098 = \underline{19.8\%} \text{ Ans.}$$

$$= 0.198$$

Numericals

V: Imp

Ques: Find two lowest permissible energy state for an electron which is confined in one-dimensional infinite potential box of width 3.5×10^{-9} m. [2020-21]

Solⁿ: The energy of particle of mass 'm' moving in one dimensional potential box of infinite height and of width 'L' is

$$E_n = \frac{n^2 h^2}{8ma^2}$$

$$E_n = \frac{n^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (3.5 \times 10^{-9})^2} = 0.49 \times 10^{-19} n^2 \text{ J}$$

$$E_n = \frac{0.49 \times 10^{-19}}{1.6 \times 10^{-19}} n^2 \text{ eV} = 0.0306 n^2 \text{ eV}$$

The lowest two permitted energy values of electron is obtained by putting n=1 and n=2,

$\therefore$ For n=1, $\boxed{E_1 = 0.0306}$ eV

For n=2, $E_2 = 0.0306 \times 4 \Rightarrow \boxed{E_2 = 0.1224 \text{ eV}}$

Ques: In a Compton Scattering experiment, X-ray of wavelength 0.015Å is scattered at 60°, find the wavelength of scattered X-ray.

Imp

Solⁿ: If λ and λ' be wavelength of incident and Scattered X-ray respectively. Then, Compton shift is given by

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

$$\lambda' = \lambda + \frac{h}{m_0 c}(1-\cos\theta)$$

$$\lambda' = (0.015\times10^{-10}) + \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos 60^\circ)$$

$$\lambda' = (0.015\times10^{-10}) + 0.012\times10^{-10}$$

$$\boxed{\lambda' = 0.027\times10^{-10}\,m = 0.027\,\mathring{A}}$$ Ans.

Ques:- X-rays of wavelength $2\mathring{A}$ are scattered from a black body and X-rays are scattered at an angle of 45°. Calculate Compton shift, wavelength of scattered photon λ'. [2018-19]

Sol^n: Compton shift, $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$

$$\Delta\lambda = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos 45^\circ)$$

$$= 0.243\left(1-\frac{1}{\sqrt{2}}\right)\times10^{-11}$$

$$\boxed{\Delta\lambda = 0.007\,\mathring{A}}$$

$$\lambda' - \lambda = 0.007\,\mathring{A}$$

$$\lambda' = \lambda + 0.007$$

$$= (2+0.007)\,\mathring{A} \Rightarrow \boxed{\lambda' = 2.007\,\mathring{A}}$$

Ques:- X-ray with $\lambda = 1\mathring{A}$ are scattered from a carbon black, the scattered radiation is viewed at 90° to the incident beam. Find the compton shift $\Delta\lambda$ and the kinetic energy imparted to the recoiling electron.

Solⁿ: Compton Shift,

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\Delta\lambda = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos 90^\circ)$$

$$\boxed{\Delta\lambda = 0.0243\,\mathring{A}}$$ Ans.

Now,

K.E imparted to recoil electron = Decrease in energy of photon.

i.e., $K.E = h\nu - h\nu'$

or $K.E = \frac{hc(\lambda'-\lambda)}{\lambda'\lambda}$

$\because \lambda' - \lambda = 0.0243\,\mathring{A}$

$\lambda' = 0.0243 + 1 = 1.0243\,\mathring{A}$

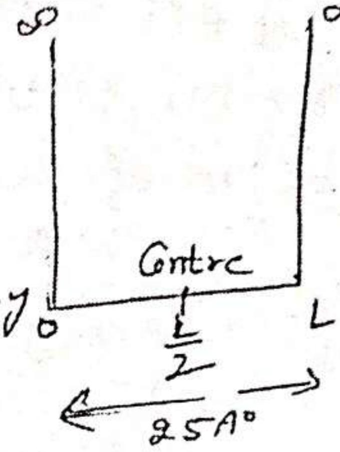
$$\therefore K.E. = \frac{6.63\times10^{-34}\times3\times10^{8}(0.0243\times10^{-10})}{1\times10^{-10}\times1.0243\times10^{-10}}$$

$$K.E. = 4.72\times10^{-17}\ J$$

or $K.E = \frac{4.72\times10^{-17}}{1.6\times10^{-19}}\ eV$

$$\boxed{K.E = 295\,eV}$$

3.- A particle is moving in one-dimensional potential box (of infinite height) of width 25A°. Calculate the probability of finding the particle within an interval of 5A° at the Centre of box when it is in its state of least energy.



Solution

we know that the wavefunction of a particle enclosed within an infinite potential well is given by

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}$$

when the particle is in least energy state $n=1$

$$\psi_1(x) = \sqrt{\frac{2}{L}} \sin\frac{\pi x}{L}$$

at the Centre of box $x = \frac{L}{2}$, the probability of finding the particle in the unit interval at Centre of box is given by

$$|\psi_1(x)|^2 = \left[\sqrt{\frac{2}{L}} \sin\frac{\pi x}{L}\right]^2 = \left[\sqrt{\frac{2}{L}} \sin\left(\frac{\pi \frac{L}{2}}{L}\right)\right]^2$$

$$= \frac{2}{L} \sin^2\frac{\pi}{2} = \frac{2}{L}\times 1 = \frac{2}{L}$$

The probability P in the interval Δx is given by

$$P = |\psi(x)|^2 \times \Delta x = \frac{2}{L}\times \Delta x$$

given $L = 25A° = 25\times10^{-10}\,m$, $\Delta x = 5A° = 5\times10^{-10}\,m$

$$P = \frac{2\times5\times10^{-10}}{25\times10^{-10}}\times100 = 40\,\%$$

Que Show that the de-Broglie wave velocity is a function of wavelength in free space.

OR.

Show that the phase velocity of de-Broglie waves associated with a moving particle having a rest mass m_0 is given by.

$$V_p = c\left[1+\left(\frac{m_0 c\lambda}{h}\right)^2\right]^{1/2}.$$

Solution According to the de-Broglie's concept of matter wave, the wavelength associated with a particle of mass 'm' moving with a velocity v is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\left(\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\right)}$$

$$\lambda = \frac{h\sqrt{1-\frac{v^2}{c^2}}}{m_0 v}$$

and the phase velocity is

$$V_p = \frac{c^2}{v}$$

From equation(1)

$$\lambda = \frac{h\sqrt{1-\frac{v^2}{c^2}}}{m_0 v}$$

$$\lambda = \frac{h}{m_0}\sqrt{\frac{1}{v^2}-\frac{1}{c^2}}$$

Substituting the value of v in terms of phase velocity from eq(2), we get.

$$\lambda = \frac{h}{m_0}\sqrt{\frac{v_p^2}{c^4} - \frac{1}{c^2}}$$

or $$\lambda = \frac{h}{m_0 c}\sqrt{\frac{v_p^2}{c^2} - 1}$$

$$\frac{\lambda m_0 c}{h} = \sqrt{\frac{v_p^2}{c^2} - 1} \quad \text{i.e} \quad \left(\frac{\lambda m_0 c}{h}\right)^2 = \frac{v_p^2}{c^2} - 1$$

$$\frac{v_p^2}{c^2} = 1 + \left(\frac{m_0 c \lambda}{h}\right)^2$$

$$\boxed{v_p = c\sqrt{1 + \left(\frac{m_0 c\lambda}{h}\right)^2}}$$

This expression shows that for a particle of mass m the wave velocity is greater than c and is a function of λ even in free space.

Que The de-Broglie wavelength associated with an electron is $10^{-12}m$. Find its group velocity and phase velocity.

Solution As the wavelength associated with an electron is extremely small ($10^{-12}m$), hence relativistic correction should be applied. The relativistic mass 'm' of an electron is given by

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{m_0}{m}$$

ie $$\sqrt{1-\frac{v^2}{c^2}} = \frac{m_0c^2}{mc^2}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{E_0}{E}$$

$$1-\frac{v^2}{c^2} = \frac{E_0^2}{E^2}$$

$$\boxed{v = c\sqrt{1-\frac{E_0^2}{E^2}}}$$

So, E_0 is rest energy and E the total energy of moving electron. The total energy of the relativistic particle is given by $$E = \sqrt{(pc)^2+(m_0c^2)^2}$$

$$pc = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^8}{10^{-12}\times1.6\times10^{-19}} = 1243.1\,keV$$

$$m_0c^2 = \frac{9.1\times10^{-31}\times(3\times10^8)^2}{1.6\times10^{-19}} = 511\,KeV$$

$\therefore$ Total energy of the electron,

$$E = \sqrt{(1243.1)^2 + (511)^2}$$

$$E = 1344.03\,\text{keV}$$

Hence $$v = c\sqrt{1 - \left(\frac{511}{1344.03}\right)^2}$$

$$v = c\sqrt{1 - 0.144}$$

$$v = 0.925c$$

Group velocity of the de-broglie waves $v_g = v = 0.925c$

Phase velocity of de-broglie's waves $v_p = \frac{c^2}{g} = \frac{c^2}{0.925c}$

$$\boxed{v_p = 1.08c}$$

5 Year's

University Previous Questions

(Questions Bank)

5 Years AKTU University Examination Questions		Unit-1	
S. No	Questions	Session	Lecture No
1	What is Wien's law?	2016-17 2017-18	1
2	State Wien's displacement law and Rayleigh-jeans law?	2020-21	1
3	Why is black the best emitter?	2021-22	2
4	Describe energy distribution in black body radiation?	2016-17 & 2021-22	2
5	Write the assumptions of Planck's hypothesis.	2018-19	2
6	Explain the modified and unmodified radiations in Compton scattering?	2016-17	2
7	What is Compton effect & Compton shift? Derive the necessary expression for Compton shift.	2016-17 2018-19 2020-21 2021-22	2
8	What is Compton effect? Derive a suitable expression for Compton shift $\lambda' - \lambda = \frac{h}{m_0c}(1 - cos\theta)$.	2018-19	2
9	What is Compton Effect? How does it support the photon nature of light?	2019-20	2
10	What is the concept of de-Broglie matter waves	2017-18	3
11	Interpret Bohr's quantization rule on the basis of de-Broglie concept of matter wave	2019-20	3
12	What are matter waves associated with a particle generated when only it is in motion?	2020-21	3
13	What is the difference between electromagnetic wave and matter wave?	2019-20	3
14	Describe the experiment of Davisson and Germer to demonstrate the wave character of electrons.	2005 & 06	3
15	Determine the de-Broglie wavelength of photon.	2018-19	3
16	Show that the phase velocity of de-Broglie wave is greater than the velocity of light.	2007-08	4
17	Derive time independent Schrodinger wave equation	2016-17 2020-21 2021-22	5
18	Obtain time dependent and time independent wave equation?	2018-19	5
19	Show that $\Psi(x,y,z,t) = \Psi(x,y,z,t)\ e^{-i\omega t}$ is a function of stationary state	2018-19	6
20	Give physical interpretation of wave function. Also explain Eigen value and Eigen function?	2016-17 2018-19 2021-22	6
21	Find an expression for the energy states of a particle in a one – dimensional box.	2017-18	6
22	A particle is in motion along a line X=0 and X=L with zero potential energy. At point for which X<0 and X>L, the potential energy is infinite. Solving Schrodinger equation obtain energy eigen values and Normalized wave function for the particle.	2018-19	6
23	Calculate the energy of oscillator of frequency 4.2×10^{12} Hz at 27° C treating it as (a) classical oscillator (b) Planck's oscillator.	2018-19	7

24	Calculate the de-Broglie wavelength of a neutron having kinetic energy of 1eV. (Mass of the neutron = 1.67×10^{-27} kg, h= 6.62×10^{-34} joule sec)	2019-20	7
25	X-rays of Wavelength 2 Å are Scattered from a black body and x-rays are scattered at an angle of 450.Calculate Compton shift, wavelength of scattered photon λ'.	2018-19	7
26	Determine the probability of finding a particle trapped in a box of length L in the region from 0.45L to 0.55L for the ground state.	2017-18	8
27	Find the two lowest permissible energy states for an electron which is confined in one dimensional infinite potential box of width 3.5×10^{-9} m	2020-21	8
28	Calculate the energy difference between the ground state and the first excited state for an electron in a one dimensional box rigid of length 25.	2021-22	8
29	Write down Schrodinger wave equation for particle in a one-dimensional box and solved it to find out the Eigen value and Eigen function.	2019-20	8